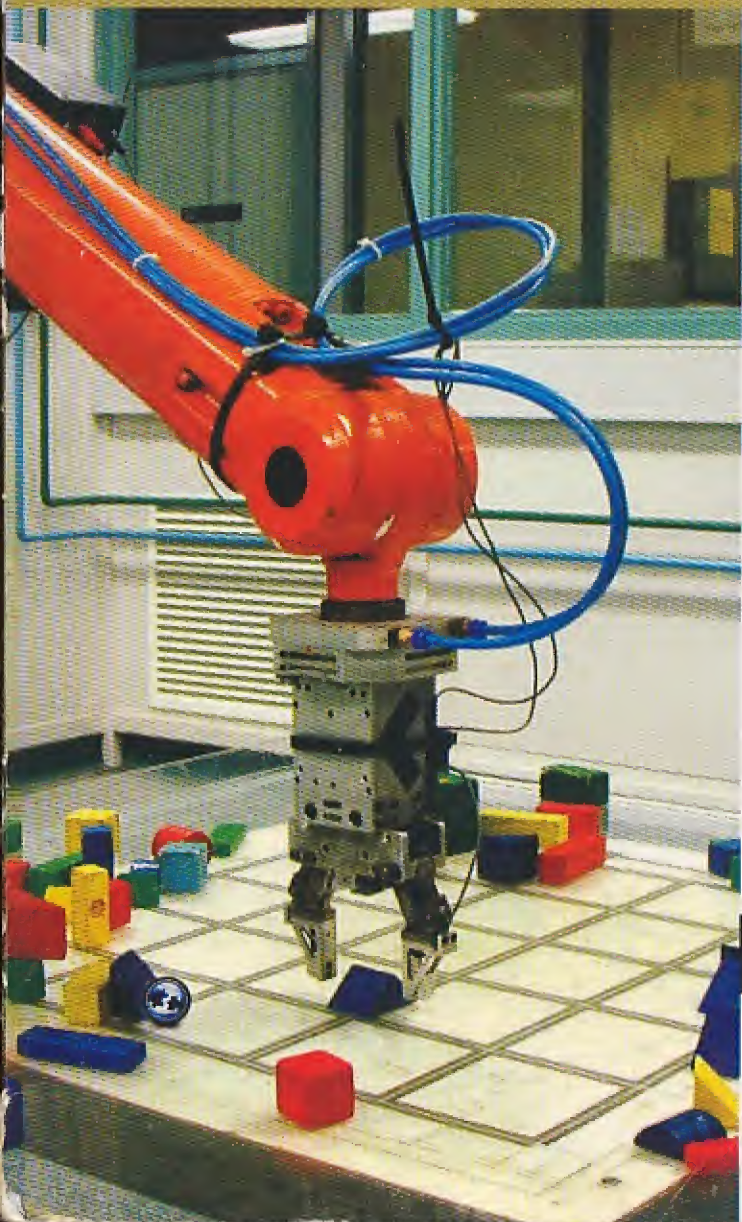
SECOND EDITION

A Textbook on

INDUSTRIAL ROBOTICS



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A TEXTBOOK ON INDUSTRIAL ROBOTICS

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Contents

Preface to Second Edition

In this edition, an attempt has been made to include some more topics of Industrial Robotics. The shortfall in most of the syllabus is taken care of to some extent. The suggestions and criticism of the readers are given due importance in revising the book in this edition. The errors have been corrected to the maximum extent. The introduction to static force analysis has been included. The topics added could make the readers to depend fully on this book for the study and information.

—AUTHOR

Preface to First Edition

This book on Industrial Robotics, deals with definitions and derivations, principles and problems studied by the undergraduate student community of Mechanical Sciences.

What, why and how of Robotics have been covered in simple and lucid terms for the students to understand the syllabus. The problems from question papers of previous years, have been dealt as a measure to build confidence among the aspiring students. The exercises with most likely theory questions and supplementary problems for practice presented at the end of each chapter invite the students and teachers to sharpen their interest to study and work out. Many reference books in this field have contributed well to the author's presentation and thought, and he likes to acknowledge it heartily. As a custom and first duty, I salute Shreemajjagadguru Shankaracharya Shree Shreemadraghaveshwara Bharati Mahaswamigalu for his divine inspiration and blessings in this effort to write this book.

My special thanks to Laxmi Publications, for their acceptance of my request to write this book. I particularly thank my wife Smt. Geeta Ganesh for her suggestions and co-operation at all stages of writing this book. Everyone who directly or indirectly extended help and favour, deserves my hearty thanks.

I invite constructive criticism and suggestions from students and readers for further improvement of this book.

—AUTHOR

1 Introduction to Robotics

1.1 INTRODUCTION

The word 'robot' has the origination in the Czech dictionary word 'robota', meaning work. The perception of a robot as given in science fiction literature in the mid-twentieth century is different from the present form of industrial robot which cannot move on its own but their physical pattern resembles the human arm. Hence the industrial robots are known as anthropomorphic 'robotic manipulators' or 'robot arms'. The industrial applications and atmospheres are diverse in nature, frequent, complex, non-reachable or harmful to human being. In all these cases the robot can be an alternative to human hands. In this advanced technological world the skill, the perfection, the productivity and the speed with which the work has to be done influence the people making decision regarding introducing a robot for manufacture and manipulate efficiently. The break even between the present practices and robotic adoption controls the investment philosophy in the Indian scenario. The entry of robotics calls for updating the education system with the basic understanding of the robot-definition, anatomy, design, control and application.

The articulated industrial robots are patterned to look like human hands with upper arm, fore arm and fingers at the end. The chest and the arms are corresponded by the links in the manipulator. The robot joints represent the shoulder, elbow and the wrist. The robot fingers are called end-effector which may be a tool or a gripper which is a holding device. The opening and closing or a movement can be by a mechanical mechanism. The movements are programmed by a computer.

1.2 DEFINITION

"A robot is a mechanical device with links and joints, guided by sensors, driven by actuators and controlled through a programmed software, to handle and manipulate parts, materials, tools and devices for performing various tasks in variety of work environments".

The definition stated above answers the question like—what type of device is the robot ?—What's are its elements ?—How is it moved ?—How is it made to function ?—what does it do ? and where can it work ? The study of the above questions appear as separate chapters dealing in detailed explanation and analysis.

The study and understanding 'Robotics' is interdisciplinary with mechanical in the domain and other streams like electrical, electronics and computer, being the supplementary and essential for the industrial robot to be flexible, efficient and accurate in operation. The links and joints are to be designed for strength and rigidity through static and dynamic force analysis. While the electric motors and hydraulic/pneumatic actuators produce robot motion.

The required positions are computed through transformations. The electronics contributes in the form of control system to closely match the desired output with the achieved output. The computer programs add flexibility for performing variety of jobs executed by the robotic manipulators. The software programs with the developed algorithms, controllers and sensing systems make the robot to posses intelligence to carry out jobs with in the work envelope, defined by the movements (degree of freedom) given to links. The motion of the links are translatory and/or rotary explaining the configuration and category of a robotic manipulator.

1.3 AUTOMATION AND ROBOTICS

When the units of production and the rate of manufacture are high the specialized machines are more useful. These machines were developed to operate faster and better, to produce mechanical and electrical automobile parts in auto-industries. Introduction of such machines lead to industrial automation which can be defined as.

"For the operation and control of production or manufacture the mechanical, electrical, electronics and computer based systems are integrated to form a technology known as Industrial Automation".

Few examples for industrial automation are

—Special purpose machine (SPM) tools.
—Computer Numerical controlled (CNC) machines.
—Transfer lines.
—Flexible Manufacturing systems (FMS).
—Robotics.

Hence it is clear that all automatic machines are not robots. But robotics is a type of industrial automation. Based on the flexibility and adoptability to volume and speed of different type of production processes the industrial production is classified into (*a*) Hard Automation (*b*) Soft Automation.

• **Comparison between Hard Automation and soft Automation**

Features	*Hard Automation*	*Soft Automation*
1. Cost effectiveness	• Good at very high production volume.	• Good for moderate production volume.
2. Flexibility	• limited	• High.
3. Life cycle	• To be for a longer period.	• For short and medium period of cycle.
4. Batch Production	• Not suitable.	• Highly suitable.
5. Control through software	• Not possible.	• Easily possible.
6. Obsolescence of the machine	• Happens with change in model for which the part is manufactured.	• Does not happen because the software can be changed.
7. Efficiency of the operation.	• Comparably high.	• Equally high.
8. Examples	• Automatic machines, special purpose machines, machines not controlled through software, Transfer lines etc.	• CNC machines, Robots and reprogrammable machines, etc.

The Fig. 1.1 summarises the qualitative comparison of cost effectiveness of manual operation, hard automation and soft automation, [4] with respect to the volume of production. The unit cost of production is minimum for volumes less than q_1. This is generally possible in cases of small batch processing. For a volume of production beyond q_2, a very high quantity, the unit cost is least, for which hard automation is most convenient.

The break-even between manual labour and soft automation, the point A, and the break-even between soft automation and hard automation, the point B, gives the range of production volume over which 'robots' are cost effective. Over the production band between q_1 and q_2 the industrial robots are more sophisticated and less expensive.

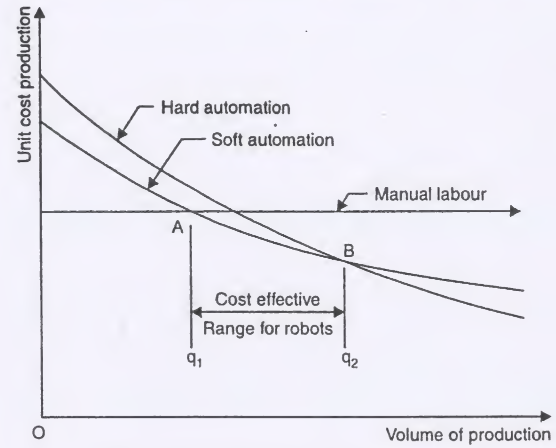


Fig. 1.1. Cost Effectiveness Comparison.

Risks of Hard Automation

- As the initial investment cost on the machine is too high, if the volume of production expected becomes to be lower, the cost per unit turns out to be higher and the profit margin declines relatively.
- The machinery specialized to produce a particular product, becomes obsolete when the design of the model is changed to cope with competition in the market.

The soft automation can be further divided into

—programmable automation, and
—Flexible automation.

The features of programmable automation are :

—The products of small batches are produced one after the other.
—The machine is reprogrammed after one product to process the further product batch.
—All product configuration are to be kept programmed and stored, and loaded in sequence for faster production.
—An industrial robot is the correct example for this type of soft automation.

The features of flexible automation are :

— Using the same manufacturing facility, different product configuration can be produced at the same instance.

— Variety of product patterns can be mixed and processed parallelly through preloaded programs.

— This possibility depends on the computational power of the computer integrated with the system.

— The ideal examples are flexible manufacturing systems, computer integrated manufacturing systems (CIMS), etc.

1.4 OCCURANCE OF EVENTS IN THE HISTORY OF ROBOTS

Year	*Type, name*	*Application Feature*	*Firm Developed.*
1959	Commercial	Controlled by limit switches and cams.	Planet corporation.
1960	'Unimate' Hydraulic drive robot.	'Programmed article Transfer' with manipulator control.	Devol's U.S. firm.
1966	Spray painting Type of Robot.	Controlled spray of paint.	Trallfa, a Norwegian firm.
1968	'Shakey'	Mobile robot with sensors and camera.	Stanford research Institute.
1971	Stanford Arm.	Electrically powered robot.	Stanford University.
1974	IRB6 robot	All electric drive.	ASEA
1974	T^3 Robot (The Tomorrow Tool)	Computer controlled robot.	Cincinnati Malicron.
1975	Sigma	Assembly operation	Olivetti "Sigma"
1978	PUMA (Programmable Universal Machine for Assembly)	Assembly operation.	Unimation by GM.
1979	SCARA (Selective compliance arm for Robotic Assembly)	Assembly operation.	Yamanashi University of Japan.
1982	RS-1 robot with robot language AML	Assembly operation.	IBM
1983	APAS-adoptable programmable assembly system	Flexible automated Assembly line.	Westinghouse Corporation.

1.5 ADVANTAGES AND DISADVANTAGES

No.	*Comparative Characteristics*	*An Advantage*	*A Disadvantage*
1.	Environmental Safety	✓	
2.	Cost constraint in investment		✓
3.	Productivity parameter	✓	
4.	Unit cost in the long run and batch	✓	
5.	Decision intelligence		✓
6.	Replacement of labour in a populated place		✓
7.	Accuracy, repeatability and work quality of new robot	✓	
8.	Stimulation multiplicity–it can work with multiple stimulli	✓	
9.	Advanced technological accessories like sensors and vision camera	✓	
10.	Degeneration of human skill and capability		✓

Health affecting environments are not hazardous to robots, where as human beings need a comfortable and safe surroundings never-the-less life supporting accessories partly take care of human activities. Air polution, noise polution and light variations do not disturb the robot activities.

Purchase cost, education and training, installation cost and maintenance make the investment decisions to be taken carefully.

The input variables and the output income units decide the productivity parameters. For a given operational time the unit output is more in case of robots compared to human labour.

Robots are more advantageous in the long run and batch-production. By changing the program robot can process the parts in batches and can produce the same for a longer period.

Handling the intelligence of a situation is art of human brain. Programming can not take care of all the possible patterns naturally embedded in the human capabilities.

In a densely populated country like India robotic adoption poses social problems leading to unemployment and labour resentment, hence replacement by robots is a management problem.

Accurate manipulability of operation and the repeatability of work are the better characteristics of robots, that make it advantageous over other machines. Quality of the jobs produced on robotic manipulators is uniform and consistence.

Human hands can process one or two ideas at a time, where as manipulators are capable of performing tasks of many stimuli at a given instance.

Advanced engineering and technological parts like sensors and vision cameras can be attached to the robot arms to resemble the human arms in carrying out the work procedure.

Skill is developed over a period of time by experience obtained by continuous work and application and determination. However robots with built in skill requires human help, which is limited to simple operations.

Although robots have superiority and certainity in some senses they lack in exactness of the responses and cannot sense the presence of undefined obstructions. Real time responses have been the limitation on the programming capabilities. The complex degree of freedom can not be tackled by the robots. Robots depend upon external source of power like electricity transformed into movements.

1.6 INVESTMENT ON ROBOT

A cost analysis and Revenue analysis.

Expenses		*Revenue*	
Direct	*Indirect*	*Direct*	*Indirect*
• Material cost	• Operational time-reduced.	• Finished goods revenue.	• Faster production revenue.
• Robot purchase cost.	• Cost of consumable peripherals-reduced.	• Labour : less requirement.	• On time supply of the product.
• Cost of programming.	• Storage cost increases due to small lot size.	• Well finished product quality.	• Increase in productivity.
• Maintenance cost.	• Installation cost is more.	• Variety of product for varies applications.	• Can work all 24 hours.
• Material storage cost increases	• Batch change over cost.	• Increase in export potential and reserve.	• Compete ahead in marketing.
• Cost of scrap.	• Materials handling cost.	• Expansion of the market leading to more sale.	• Miscellaneous.
• Tooling cost	• Building a separate work cell.	• Diversification of product range.	
	• Rework cost.		
	• Paper cost and ordering cost.		
• Remuneration	• Educating and training the operators.	• Estimated flexibility.	
	• House keeping.		
	• Estimated safety cost.		
	• Miscellaneous.		

Cost Analysis

Cost of Investment	*Operational Costs*
1. **Engineering Costs :** The cost of project planning by the company's professional staff to install a robot.	1. **Training :** This is the first operational cost which prevents displacement old workers and induction of new.
2. **Purchase Cost of Robot :** The basic price of the robot conforming to needed specification.	2. **Resources and Utilities :** The cost of energy sources like electricity, gases and hydraulic/pneumatic fluids.
3. **Cost of Installation :** Preparation of the floors and utilities—respective labour and materials.	3. **Direct Labour :** The operation of the robot work cell is included in this.
4. **Tools and Fixture Cost :** This includes the cost of end effectors, tools and fixtures to be kept in the work cell.	4. **Indirect Labour :** The labour cost not included in the direct labour. The costs like programming, supervision, set up costs are generally included in this.
5. **Miscellaneous Expenses :** The cost other than the above costs, incurred due to peripherals and display monitors.	5. **Cost of Maintenance :** The robot's part wear-out over a period of time of operation, leading to maintenance and repair of the robots. It is customary to enter into maintenance contract while purchase itself or an appointment of a separate personell to do the job.

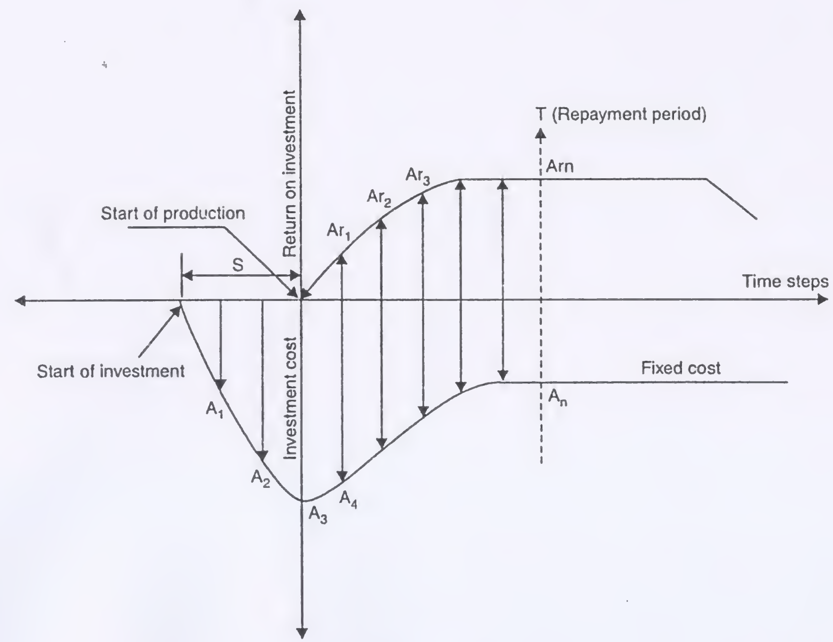


Fig. 1.2. Repayment Period Plot.

Calculation of the Repayment Period (T)

$$\frac{\begin{Bmatrix}\text{Average of the investment}\\ \text{till the break even}\end{Bmatrix}}{\begin{Bmatrix}\text{Time taken for break even from}\\ \text{the start of production}\end{Bmatrix}} = \frac{\begin{Bmatrix}\text{Average of the return}\\ \text{till the break even}\end{Bmatrix}}{\begin{Bmatrix}\text{Time taken to break even}\\ \text{from start of investment}\end{Bmatrix}}$$

$$\frac{A_i}{T} = \frac{A_r}{(T+s)}$$

and $$T = \frac{s \cdot A_r}{A_i - A_r} = \text{Repayment Period.}$$

where $$A_1 = \frac{A_1 + A_2 + A_3 + \ldots\ldots A_n}{n}$$

and n = number of steps of calculation of cost.

$$A_r = \frac{A_{r1} + A_{r2} + A_{r3} + \ldots\ldots A_{rn}}{rn}$$

rn = number of steps of calculation of return from the start of production.

s = time taken for the start of production from the date of investment.

The plot of time step against cost-investment and return value is depicted in the Fig. (1.2). The x-axis is the representative of time steps and the y-axis above the x-axis is the portion giving return on investment where as the portion below x-axis is indicator of investment-cost. The start of investment is the point on negative x-axis where as the start of production is at the origin.

The horizontal part of the investment curve is the portion showing fixed cost incurred after stabilization of production. Some where on this the repayment on investment starts. The horizontal portion on the return curve shows range of peak return on investment.

The decision of funding a potential investment project depends on the selection of a minimum-attractive-rate of return (MARR) by the company introducing robotics in its industrial setups. MARR is selected based on

- The growth potential of the market.
- The competitor's market share.
- Market survey of own company share.
- The product price and the projected profit.
- The capacity utilization.
- The introduction of a new product in the product range.

Generally MARR ranges from 20%–50%.

1.7 SOCIAL IMPACT

Replacement of the conventional machinery by industrial robots has a transforming impact on the industrial society, especially direct labour, in the form of displacement, convertion and change in the employment pattern, and also training and education system is also to be modified. The management is inclined to opt for introducing robots in the industry, biased by the derived benefits like higher productivity and flexibility. Robotics has stretched its hands in various industrial fields like assembly of parts, material handling, welding and spray painting, irrespective of environs of operation hazardous or non-hazardous in nature. The impact of robotics can be studied and analysed under the following sub headings.

• Impact of Robotics on Direct Labour

A robot performing multiple tasks, can be a substitution for more than one human workers leading to the displacement, shift of direct labour to indirect labour activities and change in strategy in the appointment of new workers. The set up of the work-space and operating the robots need the education and training the direct labour in convertion to indirect labour, which involves diversion from direct manual participation in the production work performed. The change from direct to indirect jobs is subjected removal of high degree of skill, monotony and organization of activities in the conventional work area occupied by non-robotic machines. The new workers appointed in a robot installed industries need to be knowledged in installing, programming, inspecting, trouble shooting and maintenance of the industrial robots. The knowledge content, the technological skill and the education standard of the operator has to be improvised as the expertise needed in a robotic cell has to match consistently. The job opportunities open up in the robot manufacturing industries for the upgraded technical human skills.

To counter the unrest among the workers the labour unions have to be taken into confidence through sufficient prior notice, minimum-careful-displacement, new technology adaption training and guidance, also convincing the security of job is a serious task as well.

• Professional Adjustment Impact

The professional and the semi-professional forces in a conventional industry are adjusted to age old techniques like process plans, try outs, personell management, quality control, rigorous testing procedures and material movements. With the advent in automation they have to be familiar with and expertised in computer programming, altering the software, robot maintenance, optimization of processes, system analysis, product planning etc. The engineers from specialization in machine design, machine tool technology, control system engineering, electronics and computer science can fulfill the need of professionals in the robotic industries.

• Need for Education and Training

The present standard of education has to be revised to care of the (1) need of highly educated force (2) shortage of robot technicians (3) deficiency in robot language programmers (4) good well equipped laboratory facilities and instructors (5) short fall in training institutions (6) consultants to retrain the existing task force.

1.8 LABOUR, ROBOTS AND PRODUCTIVITY

According to the management perspective the productivity enhancement is a social issue of major concern to enable the company to withstand the competition locally and globally. The productivity of a robot over the human labour, especially in the batch production industry is remarkably high and effective in the unit cost of the product. The definition and the factor contributing to productivity make the understanding better.

$$\text{Productivity} = \frac{\text{units generated}}{\text{units fed into}}$$

The units generated can easily accountable by the output of the products, the unit cost of which depends again on the productivity.

The factors contributing to decide on the units fed in are

— Capital investment on machinery.

— Technical knowledge inductment.

— The direct and indirect labour hours.

The difference in input and the output is attributed by the following factors,

- The machine idle time.
- The man idle time.
- The generation of scraps.
- The possibility of rework.
- The equipment breakdown.
- Insufficient tooling aids.

Substantial improvement over these factors can be observed by the use of robotics as the means of industrial automation providing higher productivity factor. Another aspect is that the robots can work all 24 hours in a day and throughout the week leading to the better utilization of the equipment. But also the robotic automation improves the quality and appearance of the product produced by induction of technical knowledge in the form of software control.

The change over, certainly is a important social issue as it denies job opportunity to human beings with definitely serious financial and emotional impact on the workers themselves and their families as well.

1.9 MANAGEMENT AND ROBOTICS

The decision for induction robot in an industry has to be based on certain workouts by the low level management on the directions of the high level management. The following steps can be identified to be the workout approaches.

- Technology familiarization in the beginning.
- Identification of the application potentials through plant survey.
- Suitable selection of the operation.
- Study of the robot specification to the application.
- Thorough economic analysis and capital flow pattern estimation.
- Planning and control.
- Engineering and installation.

1.10 OVERVIEW OF ROBOTS

The general understanding about the robotics applicable in industries in particular are discussed in detail in the following sub-headings.

- **Degree of Freedom :** The translatory and rotary motions of the arms of the robots.
- **The Reference Frames :** The cartesian frames attributed to the base, joints and the tip of the robot arms.
- **Robot Joints :** The type of arm connections between which different types of motions are, like linear and rotational, possible.
- **Configurations :** The total set of movements of the robot manipulators of different types generally available in the industry.
- **Robot Components :** All possible types of components used in the industrial robots, which constitute them in complete.

- **Robot Specification :** The characteristics that has to be incorporated while desining and selection of the components, also to be considered while purchasing it.
- **Modes of Programming and Control :** The modes of programs and languages to produce a controlled action in a robot for the robot to function effectively.

The selection of the above features while making and purchase decision of the industrial robots depends upon the application of the robot in the various types of jobs, which is discussed in the forth coming section sub-headings.

Degree of Freedom

A body suspended in space can have six positive degrees of freedom and six negative degrees of freedom. The three degrees of freedom are the translatory or linear degrees of freedoms along the positive cartesian axes and three along the negative cartesian axes which are opposite. Six rotary movements about the cartesian axes of which three are clockwise and remaining three are anticlockwise. The illustration of the degree of freedom is shown in Fig. 1.3.

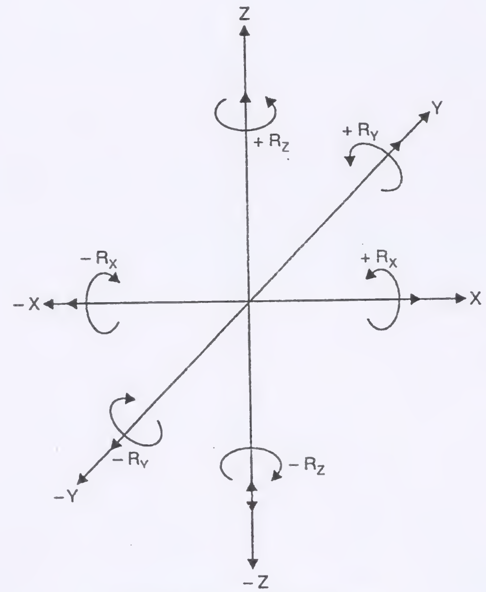


Fig. 1.3. Degree of Freedom.

The selection of the degree of freedom depends on the application to reach a specific point in the work, envelope in which the job is located.

• The Reference Frames

There are three types of reference frames attributed to the robot structure.

1. Base Reference Frame. The basic x, y and z axes are the three axes of the base. The base may be fixed or rotate about the z-axis according to the need of the application. The base reference frame is the universal frame of reference for a robot which is dipicted as in Fig. 1.4 (a).

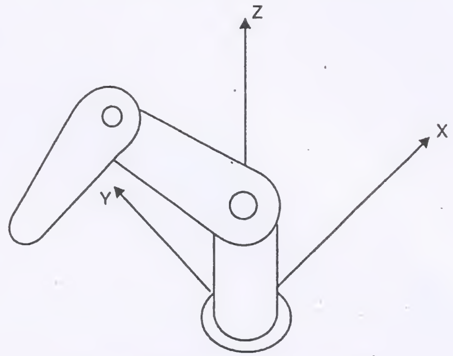


Fig. 1.4. (*a*) Base Reference Frame.

2. Joint Reference Frame. The reference axes defined at the joints of the robot, are called the joint reference frame. The joint can have both translatory and rotational movements about it defined axes. In this case the frame is not fixed. The joint frame is a shown in the Fig. 1.4 (*b*).

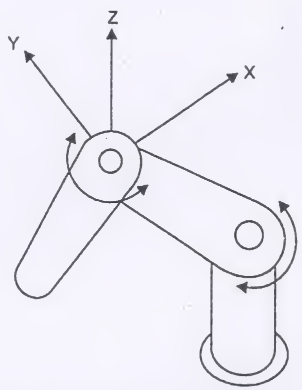


Fig. 1.4. (*b*) Joint Reference Frame.

3. Tool Reference Frame. This is the local frame of reference defined by the axes at the arm tip or the robot hand. The tip or the tool reference frame is related to the base reference frame by the transformation of the co-ordinates.

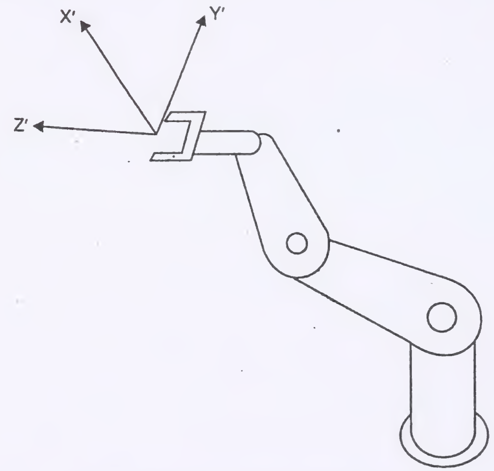


Fig. 1.4. (*c*) Tool Reference Frame.

• Robot Joints

The robot joints are made to produce motions which can be linear, rotary or spherical in nature. The joints that give linear motion are known as 'prismatic joints', and the joints producing rotary movements are named as 'revolute joints'. The joints that produce multiple rotations are 'spherical joints' which are uncommon in industrial applications.

The motion to the joints can be given by hydraulic, pneumatic or electric drives. The hydraulic or pneumatic drives can take higher payloads and for light payloads the electric drives are sufficient. The most of the industrial application robots have either prismatic joints or the revolute joints as the incorporation of the drives in these kinds of joint is easy and the control of the drives is simple.

• Robot Configurations

The possible types of movements that a robot can provide defines the configuration of a particular robot. The different configurations of different robots help in positioning of the robot hand in the defined co-ordinate of the work-envelope. If 'P' represents the prismatic joint and 'R' represents the revolute joint then a robot with three prismatic and 2 revolute joint is configured as 3P2R robot.

1. Cartesian (3P) Robot. These type of robots have three degrees of rigid body freedom. They have three prismatic joint which produces three linear motions in x, y and z directions. The illustration is given in Fig. 1.5 (*a*).

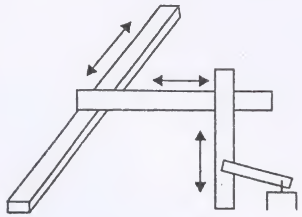


Fig. 1.5. (*a*) Cartesian.

2. Cylindrical (2 PR) Robot. This type of robots have two prismatic joints and one revolute joint. The two prismatic joints give linear movements about any two axes and the thrid movement, rotation is produced by the revolute joint. The sketch of such a robot is given in Fig. 1.5 (*b*).

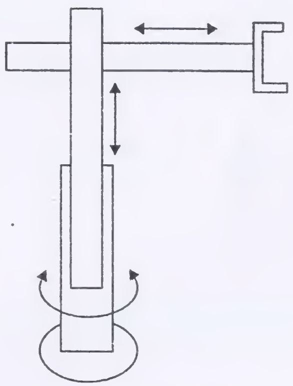


Fig. 1.5. (*b*) Cylindrical.

3. Articulated/Anthropomorphic Robot (3 R). The robots of this type have three revolute joints giving three rotary movements resembling the human hand.

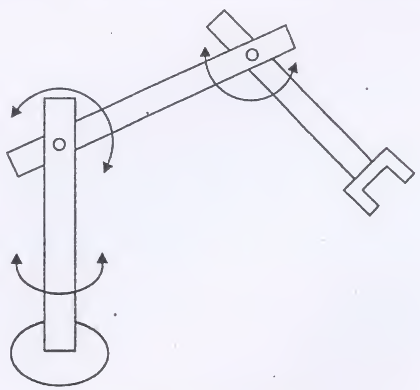


Fig. 1.5. (*c*) Articulated.

• **Spherical (2 RP) Robot.** Two revolute joints and one prismatic joint characterize this type of robot in which there are one linear and two rotary movements produced at the joints. Refer Fig. 1.5 (*d*) for the illustration.

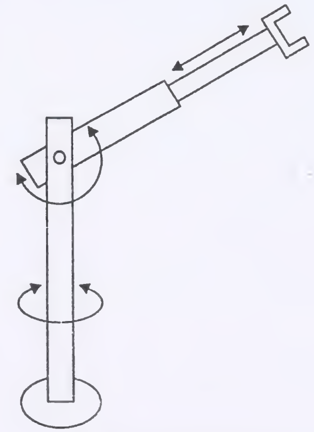


Fig. 1.5. (*d*) Spherical.

• **SCARA (Selective Compliance Assembly Robot Arm).** This is a specially configured robot which has two horizontal and parallel revolute joints with the axis vertical and one prismatic joint which can move the arm vertically up and down. This finds use in assembly operations. The Fig. 1.5 (*e*) shows schematic of the SCARA robots.

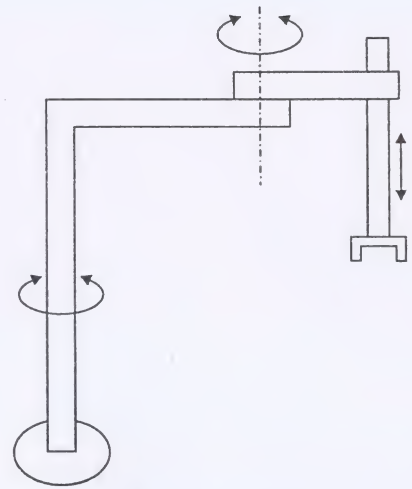


Fig. 1.5. (*e*) SCARA.

• Robot Components

For an industrial robot to carry out the assigned task of its capability, it has to have certain parts and accessories as listed and explained below.

1. Arms. The various links and joints make the anatomy of the robot which are also known as manipulators. Manipulators are the mechanical parts of a robot.

2. End-effectors. The part that is connected to the end of the arm constituting the robot hand is the end-effector which in itself is different for different applications, like spray coat gun, a welding electrode hodler, part gripper, glue applying device or a special purpose tool.

3. Actuators. The joints of the robots are powered by what are known as actuators, that produce, rotary or translatory movement in the links. The power delivering systems can be hydraulic or pneumative drives, and servo-motors or stepper motors which are direct drive types.

4. Sensors. These are the parts that recognise the robots position while in movements or when static. The further movements or action depend on the feedback of the information collected by the sensors. Sensors also perform the function of gathering data about it surrounding (work cell) which aids in processing the task. The touch and tactile sensors, vision system, force sensors, speech processors are some of the examples to the sensors.

5. Controllers. These are electronic devices which manipulate the signals from the sensors, to be provided to the drives in a understandable pattern to produce actions. The action produced may be matching or may not be tallying with the desired output. The deviation is fed back in the form of an error which adjusts the reference input to a actuating signal. The control elements and the feed back elements constitute the control system.

6. Software and Hardware. A computer software and hardware process through computation of the symbolic codes to derive the needed purpose of position, speed and accuracy of motion obtained by the kinematic equations. The monitors the peripherals and computer systems are the hardware parts. The programming languages can be a low level language like machine language or a high level language like the present day high-level languages.

The robots can be made out of the above mentioned components designed and selected to suit the derived specifications to fulfill the needs of a particular industrial tasks of material handling, welding, painting, gluing and assembly tasks, in spite of that the present day robots copy the functions and actions performed by the human beings.

• Robot Specifications

Any standard product has to be designed and marketed under certain requisite specifications or characteristics which aids in making decision to select and categorize it. Definite specifications that an industrial robot should bear are the maximum load carrying capacity, the repeatability or accuracy, the precision and the maximum and minimum reach defining the work space.

+ Payload : The rated load carrying capacity of an industrial robot is defined by its weight of the object or the tool held by the gripper, without affecting other functional characteristics like allowed tip deflection, control of motion along defined path etc. The overload may lead to the malfunctioning of the robot systems.

+ Repeatability : The accuracy with which the particular defined position can be repeatedly achieved by a robot is the repeatability. To arrive at the repeatability of a robot the statistical procedure of distribution of the positions have to be recorded and analysed and the estimated error has to be adjusted through programming the repeatability is affected by the

condition of the robot components also. The error in robot positions can give a random picture also, which has to be defined by the experimentations.

+ Precision : The reach of a position of a robot is defined by the resolution of the actuators and the control feed back systems. The robots precision is given in length units.

+ Reach : The lengths of the lengths of the links the configurations define the reach of an industrial robot. The maximum and the minimum extents of the robot positions give an idea about the reach of the robot, which is also useful in the specification of the work-envelope of the robot.

• Modes of Programming and Control

The instructions for path and position controls are provided through different programming languages for different robots manufactured by different companies. The instructions are the codes which vary from low level machine languages to high level languages understandable easily. The optimal paths to be followed to achieve the desired positions through the control system actions are coded and recorded in the form of software. In the process of development the following robot languages are generated for the purpose of programming.

APT	—	Automatically Programmed Tooling.
AL	—	Assembly Language
VAL	—	Victor's Assembly Language
AML	—	A Manufacturing Language
MCL	—	Manufacturing Control Language.
AUTO PASS	—	An IBM Language.

1.11 THE CHARACTERISTICS AND APPLICATION OF THE PRESENT ROBOTS (INDUSTRIAL)

Features → / *← Application*	*Degree of Freedom*	*Structure*	*Drive System*	*Program*	*Control System*
Material Handling	3–5	Jointed arm	Pneumatic or Hydraulic	Manual or Powered lead through	Limited sequence or point-to-point playback
Machine Loading and Unloading	4–5	Polar, Cylindrical, Jointed arm	Electric or Hydraulic for (Heavy pay loads)	Powered lead through	Limited sequence or point-to-point playback
Spot Welding	5–6	Polar, Jointed arm	Hydraulic or Electric (light)	Powered lead through	Point-to-point playback
Arc Welding	5–6	Polar, Cartesian, Jointed arm	Electric or Hydraulic	Manual or Powered lead through	Continuous path playback
Spray Painting	6 or more	Jointed arm	Hydraulic	Manual lead through	Continuous path playback
Assembly line	3–6	Jointed arm, Cartesian, SCARA.	Electric	Powered lead through or textual language	Point-to-point or continuous.

1.12 ADVANCED TECHNOLOGICAL FEATURES OF A MODERN ROBOTS

- Multiple adaptable robotic arms with modular construction.
- Multiple nodes with one controller assisted by temperature gauge, pressure gauge and position sensors.
- **Motion Control :** By mechanical couplings (coupled motion control) and co-ordinated kinematics and dynamics.
- Large work envelopes and higher payloads managed and controlled with servo tuning to avoid resonance and vibration.
- Usage of micro-controllers and embedded systems for less power requirement, compact in size, changeable functions, less movable parts for longer life, chip forming the brain.
- Controller Area Network (CAN) connections.
  - — controls efficiently the distributed intelligence.
  - — good price-performance ratio.
  - — give reliability through error detection and error handling system.
  - — with immunity against electromagnetic interference.
  - — giving dynamic connection and disconnection of nodes for flexibility.
  - — providing real time capability for better repeatability, accuracy and precision.
  - — **Communication :** Radio frequency and infra red links for digital communications.
- Programmable Automation Control (PAC) for rapid advancement in capability for which re-engineering is needed, good portability of control engine.
- **Robot Vision :** Machine vision replaces human vision through video cameras, special computer hardware and software.

1.13 NEED FOR ROBOTS

- Accuracy aspect : The robots can perform tasks with highest accuracy, repeatability and the finish is of high quality.
- Environmental aspect : They can operate under the environments hazardous to human being.
- Human aspect : The human error is eliminated by use of robot. Human beings cannot work round the clock without fatigue.
- Skill aspect : The robots controlled by computer program can execute the tasks with better skill than human being.
- Performance aspect : Productivity is enhanced by induction of robots. They can produce better performance and efficiency than human being.
- Automation aspect : The highest technology component of automation in robots can give a competitive edge in the international level.

1.14 THE CHARACTERISTICS AND APPLICATIONS OF FUTURE INDUSTRIAL ROBOT

Features → *↓ Application*	*Degree of Freedom*	*Structure*	*Drive System*	*Program*	*Nature of Task*	*Control System*
• Material handling	3–5	Jointed adaptable robot arm	Servo motors	Programmable automation control (PAC)	Safe/hazardous complicated	Motion controllers with sensor technology.
• Part loading and unloading	4–5 Multiple arms	Polar, cylindrical, Jointed arm (Adaptable)	Electronic, Servo motors (For heavy payloads)	Programmable automation control (PAC)	Complicated and safe environs.	Micro controllers and Motion controllers with vision.
• Spot (Tack) Welding	5–6	Polar, Jointed adaptable robotic arm	Electronic stepper Motors.	Programmable Logic controllers (PLC)	Simple and safe.	Micro controllers with changeable functions.
• Arc Welding	5–6	Polar, modular cartesian with adaptable jointed arm.	Direct drive servo motors	Programmable automation control (PAC)	Complicated and unsafe.	Continuous path motion controllers with sensor technology.
• Spray Coating	6 or more	Jointed arm with adaptable gun	Hydraulic actuators	Programmable Logic Controllers (PLC)	Simple and unsafe.	Continuous path motion controllers.
• Electronic Assembly	3–6 Multiple arms coupled motion.	Jointed adaptable, cartesian modular robotic arm.	Stepper motors and direct drives	Programmable Automation Control with Controller area Network (CAN)	Complicated and safe.	Micro controllers, nodes with sensors and end effectors with vision.

- **Application of Robots :**

1. **Material handling.** The jointed arm robots with 3–5 degrees of freedom can serve the material handling application. Hydraulic or pneumatic drive with manual or powered lead through teaching would give motion in the present robot designs. The next generation robots are expected to use servo motors with Programmable Automation Control (PAC). In the future robot motion controllers with sensor technology would replace the point-to-point and sequence control action. They are expected to be used in both safe and hazardous environments.
2. **Machine loading and unloading robots.** Polar (P2R) robots, cylindrical (2PR) robots jointed arm (3P) robots with 4–5 degrees of freedom are used for such application. Electronic and servo drives are the future trends in the drives as compared to present electrical and hydraulic drives. PAC can replace the powered

lead programs. Micromotion controllers with vision can be control system of next generation rather than the present point-to-point or limited response systems. They find application in both safe and diverse atmospheres.

3. **Welding.** Welding classified into spot welding and continuous welding. Polar, cartesian and jointed arm robot with 5–6 degrees of freedom are presently used. Similar robots with adaptable arms can replace in future. The Programmable Logic Controllers (PLC) are replacing the earlier manual and powered lead through robots. The hydraulic and electric drives are replaced by stepper and servo motors for higher accuracy. Point to point and continuous path are being controlled by micro motion controllers with sensor technology. Simple, safe, complicated and unsafe environments are to be considered.
4. **Spray painting.** The jointed arm robots with 6 degrees of freedom having hydraulic drive are operated in continuous path controlled by manual lead through, as seen in present robots. Adaptable arm and PLC controls operating in complicated and unsafe atmosphere are future trend.
5. **Assembly robots.** Jointed arm, cartesian, SCARA robots with 3–6 degrees of freedom having electric drive and powered lead through control are the seen features in robots. Use of adaptable modular arm with coupled motion using PAC and CAN programs operating in unsafe atmospheres controlled by micro controllers, sensors and vision system having arms driven by stepper and direct drive are advanced features being incorporated.

EXERCISE

1.1. Define a Robot. *(VTU–Jan./Feb. 2003)*
1.2. Discuss the advantages and disadvantages of using robots in industry. *(VTU–Jan./Feb. 2003)*
1.3. Explain the relationship of robotics with industrial automation and illustrate the same with a suitable example. *(VTU–Jan./Feb. 2004)*
1.4. Enlist the choronology of developments related to robotics technology until 1990. *(VTU–Jan./Feb. 2004)*
1.5. What are the costs involved in the calculation of pay back period for investment made on robot? *(VTU–May/June 2004)*
1.6. What are the advantages of using robots in industry ? *(VTU–May/June 2004)*
1.7. What is industrial automation ? What are its types ?
1.8. Compare hard automation with soft automation.
1.9. Explain the cost-effectiveness of using robots in industries with an illustration.
1.10. Explain the features of programmable and flexible automation in industry.
1.11. Explain the cost and revenue analysis of investment on robotic implementation in an industry.
1.12. Explain the calculation of repayment period on the implementation of robotics, with a cost-return plot.
1.13. What is MARR ? What are the basis on which MARR is selected ?
1.14. Discuss the impact of robotic induction on direct labour.
1.15. How are the professional staff adjusted to the impact of robotics ?
1.16. What are the needs of revised education system to take care of induction of robots in industry ?
1.17. What is productivity ? What are the factors that assigns variation to productivity in an ordinary industry as compared to robotics ?
1.18. What are the look-outs of management implementing robotics in an industry ?

1.19. Give a brief overview of robots.
1.20. What are various types of reference frames attached to a robotic structure ? Explain with example.
1.21. Briefly discuss the various robot components.
1.22. Enlist the characteristics and applications of presently used industrial robots.
1.23. What are the advanced technological features of modern robots ?
1.24. Enlist the applications and characteristics of future industrial robots.
1.25. Explain the application of robots with the features available at present and the features of future trend.
1.26. State and explain various aspects that justify the needs of robots in industry.

2 Structure of Robotic System

2.1 ANATOMY OF A ROBOT

The industrial robots resemble the human arm in its physical structure. Like the hand attached to the human body the robot manipulator or robot arm is attached to the base. The chest, the upper arm and fore-arm in the human body compare with the links in the robot manipulators. The wrist, elbow and the shoulder in the human hand are represented by the joints in the robot arm. As the industrial robot arm compares with the human hand, they are also known as "anthropomorphic or articulated robots",

Anatomy		*Representation*
1. Body	⟶	Base
2. Chest	⟶	Link
3. Shoulder	⟶	Joint
4. Upper arm	⟶	Link
5. Elbow	⟶	Joint
6. Fore-arm	⟶	Link
7. Wrist	⟶	Joint

The drives or motion to the links is provided at the joints. The joints motions can be rotational or translatory (sliding). The tool known as end-effector (gripper) is attached to the

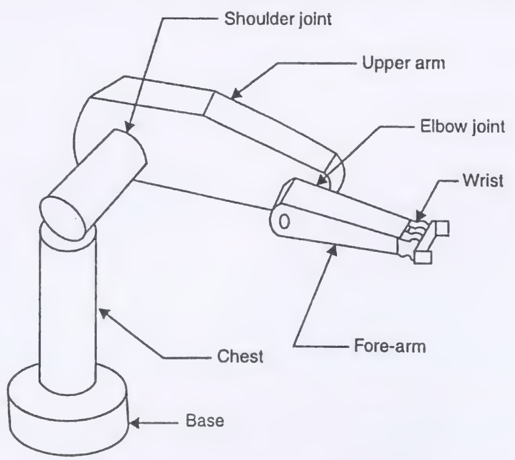


Fig. 2.1. Robot Anatomy.

wrist. The end-effectors are not considered as a part in the robot anatomy. The robot anatomy is shown in Fig. 2.1.

2.2 CLASSIFICATION OF ROBOTS

According to Japanese Industrial Robot Association (JIRA)	*According to Association Francaise de Robotique (AFR)*
1. **Class 1: Material handling device:** A man operated multiple degree of freedom machine used for handling materials.	1. **Type A:** Manual handling devices
2. **Class 2: Fixed Sequence Robot:** Robots performing unchangeable pre-determined stages of work in a sequence (fixed).	2. **Type B:** Automatic handling devices with fixed cycles.
3. **Class 3: Variable Sequence Robots:** Same as the class 2 but the sequences can be easily altered.	3. **Type C:** Programmable with point-to-point or continuous path controlled by servo-motors.
4. **Class 4: Playback Robot:** The motions are taught by the operator which is recorded and then the path is followed by robots.	4. **Type D:** Programmable intelligent robots that can work gathering information from the surrounding.
5. **Class 5: Numerically Controlled Robot:** The operations are programmed by the operator using APT and given to robot.	
6. **Class 6: Intelligent Robots:** These type can understand tasks in a surrounding and act intelligently.	

2.3 ROBOT CONFIGURATIONS

Sl. No.	*Configuration*	*Figure Number*	*Number of Revolute Joints*	*Number of Prismatic Joints*	*Example*	*Type of Work Space*
1.	Cartesian Robot (Gantry Robot) (3 P)	Fig. 2.2. (*a*)	Nil	Three (x, y and z)	IBM's RS–1 robot	Rectangular
2.	Cylindrical Robot (R 2P)	Fig. 2.2 (*b*)	One	Two	GMF's M–14 robot	Cylinder shape
3.	Polar Robot (Spherical Robot) (2 RP)	Fig. 2.2 (*c*)	Two	One	Unimate 2000, Maker–110	Spherical shape
4.	Jointed Arm Robot (Articulated Robot) (3 R)	Fig. 2.2 (*d*)	Three	Nil	Cincinnati Milacron T3	Irregular space

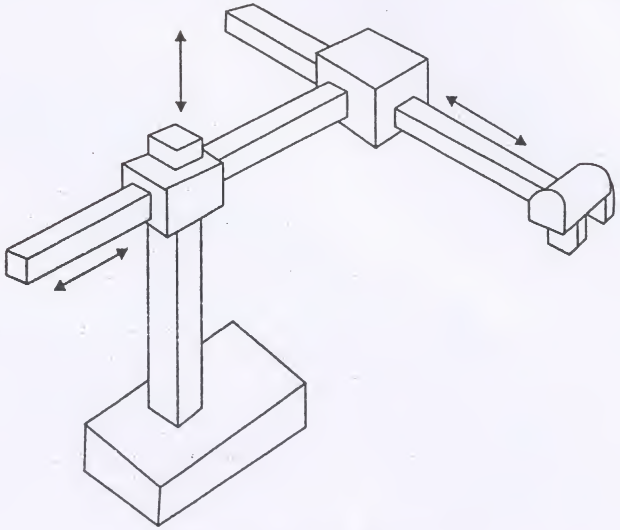


Fig. 2.2 (*a*) Cartesian Robot.

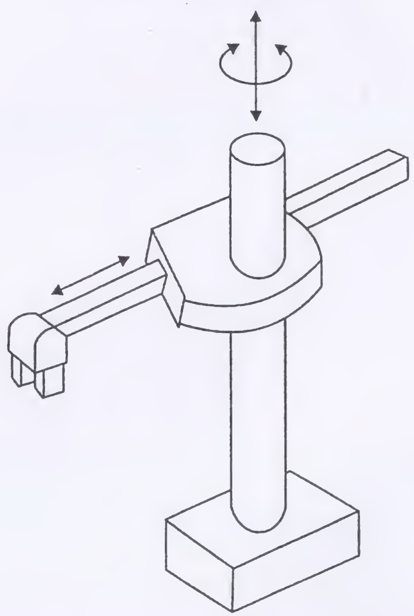


Fig. 2.2 (*b*) Cylindrical Robot.

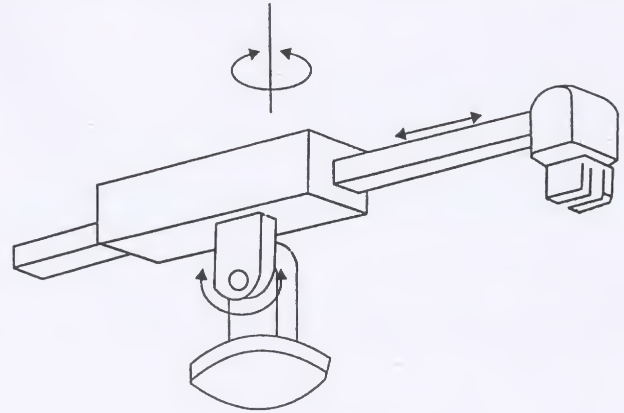


Fig. 2.2 (*c*) Polar Robot.

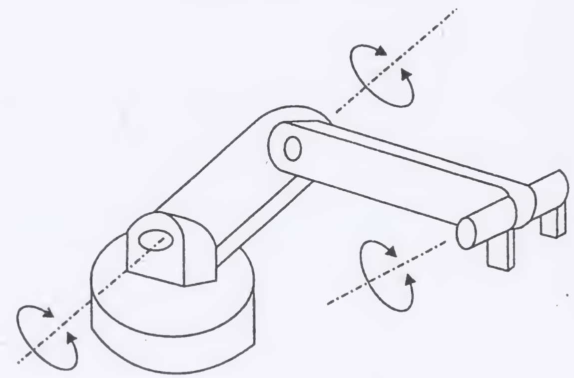


Fig. 2.2 (*d*) Jointed Arm Robot.

2.4 COMPARATIVE ADVANTAGES OF DIFFERENT CONFIGURATIONS

Sl. No.	*Configurations*	*Advantages*
1.	Cartesian Robots	• Rigid structure of box frame type • High repeatability with least error • High load carrying capability.
2.	Cylindrical Robot	• High rigidity of the manipulator • Higher load carrying capacity • Geometrical advantage in specification.

3.	Polar Robot	• Higher reach from the base • Geometric advantage in specification • Machine loading applications need this type.
4.	Jointed arm or Articulated Robot	• Higher reach from the base • Useful in continuous path generation, applied to spray painting and are welding • Reaching the conjested small openings without interference.

2.5 ROBOTIC SYSTEM

The basic functions of an industrial robotic to handle the parts and tools for operations. Robots can be made to think and act on the specific task in a work environment, to provide soft automation to the given job. The tasks are accomplished by a robotic system which is an integration of systems like mechanical system, electrical system, electronic system and computer system.

The various systems that constitute a robotic system can be categorized as follows.

- Manipulator system
- Drive system
- Control system
- End effectors
- Sensors
- Vision system
- Computer software and hardware.

The links and joints, the end-effectors are the mechanical systems, the direct drive systems are generally electric. The control system, sensors and vision system fall under electronic system. The computer system are represented by software programs and computer hardware/ peripherals.

2.6 ROBOT LINKS

The two adjacent joint axes of a robotic manipulator are connected and defined by a rigid body called 'link', which maintains a fixed relationship between the two joint axes through a kinematic function. The relationship is described by two variables—the length of the link, 'a' and the twist of the link, 'α'.

The links are numbered starting from the fixed base of the manipulator, which is called link zero (0). The first moving rigid body is link 1. Between the joint axes i and $(i-1)$ the link is numbered $(i-1)$. A general link is represented in Fig. 2.3.

• Design Considerations of a Link

A robot link is attributed by the following design considerations—

- Strength and stiffness of link.
- The material used for fabrication.

- The weight and inertia.
- The location of the bearings.
- The selection of the type of bearing.
- The fits and tolerances in the joint.
- The external shape and aesthetics.
- The friction and lubrication.

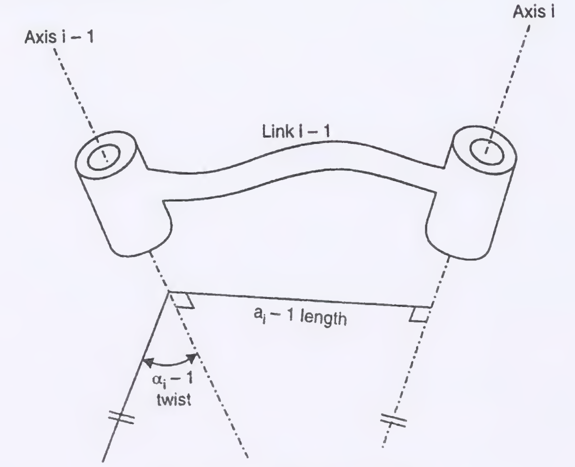


Fig. 2.3. Link Representation.

2.7 JOINTS IN ROBOTS

The relative motion featured by the sliding action between the two surfaces describes the characteristic connection known as the 'lower pair'. The lower pair formed between two links is termed as joint. The motion in the joint can be traslatory (linear/sliding) or rotary/ rotational, about or along the cartesian axes. The joints can exhibit one or more relative motion/s at a time, depending on that they are classified into following categories.

Joints	*Motions*	*Degree of Freedom*	*Figure*
• Revolute Joint	rotary motion	one	Fig. 2.4 (*a*)
• Prismatic Joint	sliding motion	one	Fig. 2.4 (*b*)
• Cylindrical Joint	one sliding and one rotary motions	two	Fig. 2.4 (*c*)
• Planar Joint	two sliding and one rotary motions	three	Fig. 2.4 (*d*)
• Screw pair	one translatory and one rotary motions	two	Fig. 2.4 (*e*)
• Spherical joint	Three rotary motions	three	Fig. 2.4 (*f*)

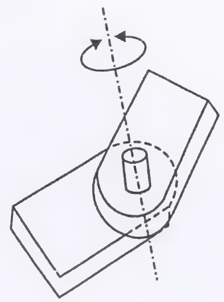


Fig. 2.4 (*a*) Revolute Pair.

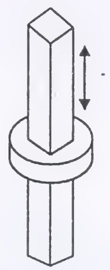


Fig. 2.4 (*b*) Prismatic Pair.

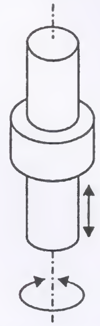


Fig. 2.4 (*c*) Cylindrical Pair.

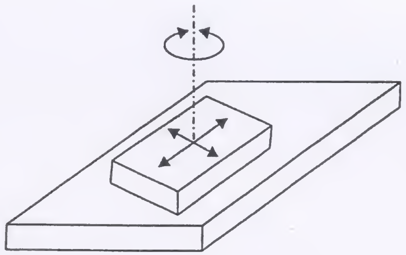


Fig. 2.4 (*d*) Planar Joint.

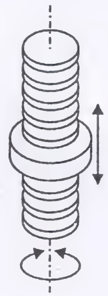


Fig. 2.4 (*e*) Screw Pair.

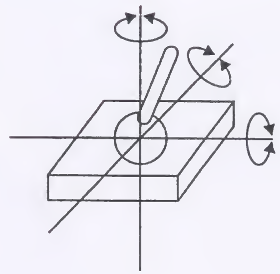
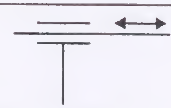
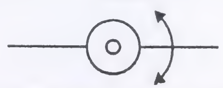
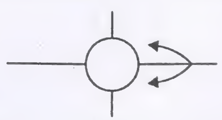


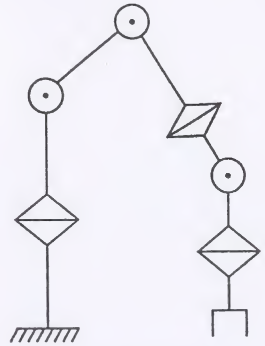
Fig. 2.4 (*f*) Spherical Joint.

2.7.1 Joint Symbols

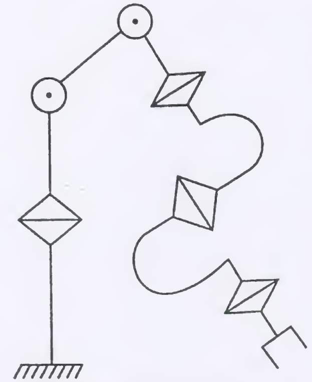
Joint description	*Symbols*
1. Prismatic joint	
2. Axial revolute joint	
3. Normal revolute joint	
4. Back and forth rotation joint	

2.7.2 Functional Diagrams of Robots

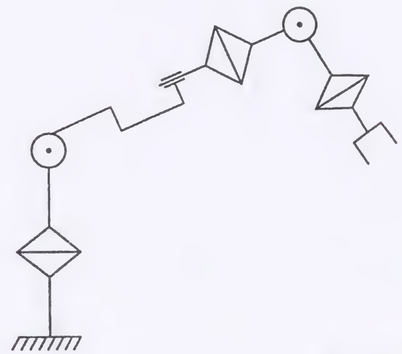
1. PUMA Robot



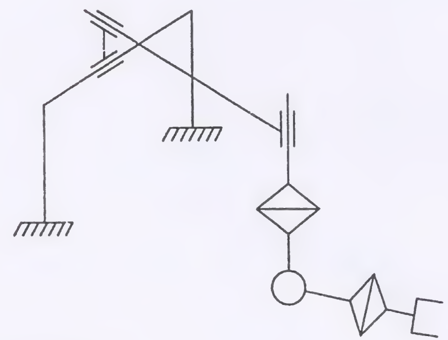
2. TR3-786 Robot



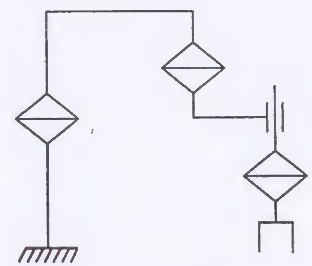
3. Kawasaki Unimate



4. IBM-7565 Robotic System



5. SCARA-Type Robot



2.8 ROBOT SPECIFICATIONS

The broad classification of the robots is conveniently based on drive system types, work space geometries and movement control techniques. Apart from these there are specific char-

acteristics provided to the customer, useful in the selection of the robotic manipulators, precisely to the required application, which are enlisted as below in the table 2.1.

Table 2.1. Robot Features

Specifications	*Dimensions*
1. Number of Axes	—
2. Capacity	kgf
3. Speed	mm/sec
4. Reach and stroke	mm
5. Operating environment	—
6. Tool orientation	deg. or radians
7. Performance parameters	mm.

• Number of Axes

The translatory movements of the links along a particular direction and/or rotational motions about a specific axis decides on the number of axes attached to a given robotic manipulator. To achieve arbitrary position for the wrist and any specific position for the tool or the gripper the general axes for the robotic manipulator are given as under, in table 2.2.

Table 2.2. General Robotic Axes

Type of axis	*Axes*	*Arbitration*
Major	1 to 3	Positioning the wrist
Minor	4 to 6	Orienting the gripper
Redundant	7 to i	Preventing obstacles

The movements assigned to links, aiding in positioning the wrist are the major types of axis which vary from 1 to 3, which can be regarded as the independent axes of motion.

Activating the tool and griper fingers are the function of the mechanisms the movement of which are not considered to be about/along independent axis which are called minor types and they vary from 4 to six axes.

The obstacles within the work envelop are to be tackled by one or more redundant axes assigned to the redundant manipulator links. The incorporation of the redundant axes adds extra complexity to the design of robot mechanism.

• Capacity

"It is nothing but load carrying ability of the robot with the allowed deflection of the manipulator end". Capacity is dependent upon the synthesis of the manipulator dimension based on statics and dynamics of the forces coming on the manipulator. The selection of a particular robot for a given application should be just enough to the required capacity rather than to go in for additional specification.

• Speed

"It is the distance moved by the tool tip in unit time". The time required to execute periodic motion while performing work, can be one of the meaningful measure of speed. Some time the accuracy with which a task is to be performed may over ride the speed. The higher

speed may be a requisite in high volume production. Higher speeds put a limit on the capacity of the robot.

• Reach and Stroke

The reach and the stroke are the measures of the dimensions of the work volume. They can be horizontal and vertical in the sense of movements. The respective reach and stroke are given in the Fig. 2.5 for a cartesian robot which has a cubical or parallelopiped work volume. A general relation between stroke and the reach is given by

$$\text{Reach} \geq \text{Stroke}$$

The equality of reach with stroke is a remote possibility in practicle situations.

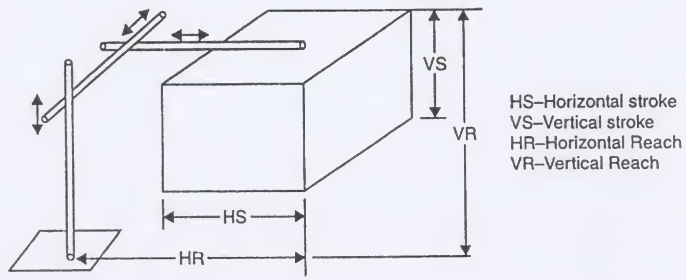


Fig. 2.5. Stroke and Reach.

• Operating Environment

The nature of the work performing surrounding of a particular robot is specific to an application. The application of robot to a job can have following types of operating environments

+ Dangerous to human beings.
+ Unhealthy in nature.
+ Harsh and difficult to access.
+ Complex and contaminated.
+ Extremely clean and dustless.
+ Ordinary and workable.

The examples of applications are movement of nuclear materials, spray coating or painting, welding (spot/continuous), loading and unloading, handling the electronic components and assembly of parts.

• Tool Orientation

The minor axes of movements determine the assumed orientation of the tool or gripper within the work envelope described by the major axes of motion. One of the tool orientation conventions is to specify the yaw-pitch-roll (YPR) of the end-effector or tool as is attributed to the aircraft movements.

The oscillating movement about one of the transeverse axis, *i.e.*, x-axis is the 'pitch'.

The rotation about the other transeverse axis, *i.e.*, y-axis is the 'yaw'.

The rotation of the tool about longitudinal axis, *i.e.*, z-axis is the 'roll'.

The tool orientation conventions are shown in Fig. 2.6.

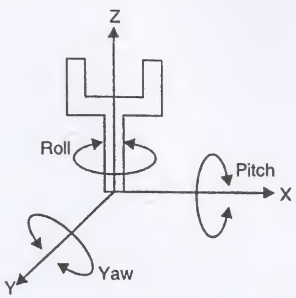


Fig. 2.6. Tool Orientation.

2.9. PERFORMANCE PARAMETERS

The manufacturing constraints and the design inevitability put some limitations on the performance of the robots. Such parameters are accounted by defining the following terminologies.

- Repeatability
- Resolution or precision
- Accuracy.

• "**Repeatability** measures the ability of the robot to position the tool tip in the same place repeatedly". The position of the tool tip in the defined work space is programmed by understable commands. In the repeatitive work sequence the tip of the tool may or may not return back to the same point, leading to a repeatability error attributed to objective and subjective inaccuracies of the robot manipulator components.

The statistical distributions of the tip return points are conceptualised and illustrated in the Fig. 2.7. The desired position of the point in space is denoted by 'D' and the achievable point through programming is 'A'. In the cycle of movement of the robot arm, the robot tries to return to a point nearest to 'A', denoted by R. The point 'R' can not be returned always to lead to the definition of the repeatability. This forms a cluster of repeated and non-repeated points distributed around the point A. The difference in position of 'A' and the point 'R' is known as repeatability error.

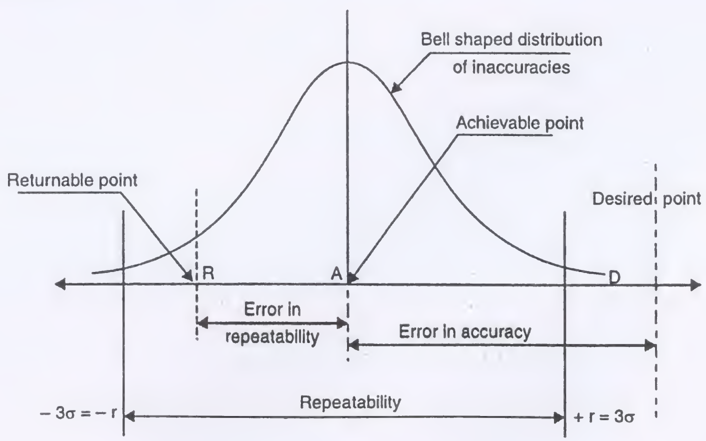


Fig. 2.7. Repeatability Representation.

The sphere of distribution in three dimensional space, of the point 'R' around the point 'A' gives the measure of repeatability. The manufacturer specifies the repeatability as the radius of the sphere on both sides of the centre of the sphere, *i.e.*, the point 'A'. If r_p is the radius of the sphere of distribution of point R from 'A' then,

$$\text{Repeatability} = \pm r_p.$$

• Resolution

"Also known as spatial resolution is the least count of the movement into which the robot's work envelope can be divided to represent the incremental or decremental steps".

The spatial resolution can be contributed by two components.

+ The control resolution.

+ The mechanical resolution.

The Control Resolution

This component depends upon the type of position control system and its feed back control elements. The hardware capability of the controller determines the control resolution, the ability to divide to motion range in to movement steps of the joints. The increments are also known as "Addressable points" depends on bit storage capacity of the control memory. The number of increments made equal by the designer, also called addressable point is,

$$i = 2^k \qquad \text{...(2.1)}$$

where k = number of bits in the control memory.

The mechanical resolution: is the other component of resolution which puts limitation on the lower limit of the spatial resolution. The following are some of the factors that contribute to the limitations.

1. Inaccuracies in the dimensions of the links and joint components.
2. Elastic deflection of the structural members.
3. Backlash in the meshing gears of power transmission.
4. Streching of the transmission components.
5. Leakage of the fluids of hydraulic/pneumatic actuators.
6. In accuracy magnification due to scaling.
7. Load handled and speed to be achieved.

When the above said mechanical factors become dominant the increase in bit capacities of memory cease to benefit much on the improvement in the spatial resolution. The maintenance condition and the age of the robot also play significant role in the process of usage of robot and determining the resolution of the robot depreciation subject.

If 'S' is the stroke of a linear, prismatic link and 'i' the number of increments or the addressable points, the control resolution in total is given by

$$R_t = \frac{S}{i} = \frac{S}{2^k} \qquad \text{...(2.2)}$$

The angular control resolution of a revolute joint can be expressed in terms of the 'n' number of slots around the circumference of the circular disk in a reductionless drive as,

$$A_c = \frac{2\pi}{n2^k} \text{ radians/count} \qquad \text{...(2.3)}$$

If the speed reduction occurs between driver shaft to load shaft through a gear train, then

$$A_c = \frac{2\pi}{nz\,2^k} \qquad \text{...(2.4)}$$

where gear reduction ratio is z : 1.

For a partial rotation of angle ϕ degree

$$A_c = \frac{\phi}{2^k} \text{ degrees} \qquad ...(2.5)$$

The spatial resolution represented with the mechanical components due to inaccuracies is illustrated by a statistical distribution is shown in Fig. 2.8.

If σ is the standard deviation about the mean point of the programmed position of tool tip, the spatial resolution takes the form

$$R_s = R_t + 6\sigma \qquad ...(2.6)$$

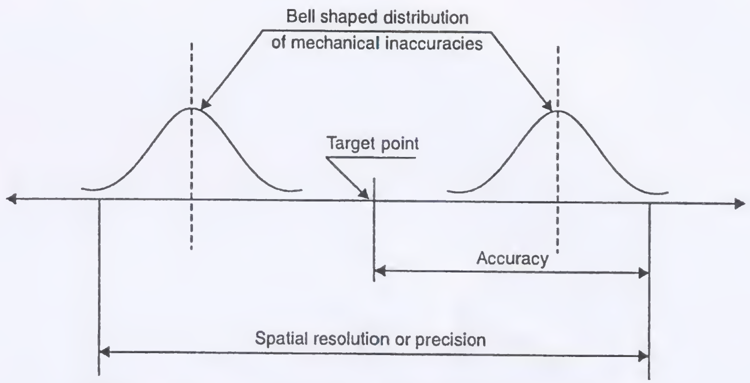


Fig. 2.8. Representation of Spatial Resolution and the Accuracy.

The spatial resolution of a particular joint in a robot arm is due to the specifications of the drive system position sensors and power transmission mechanism like gears, sprockets, chains, belts and cables. The programmed reference signal of the controlling computer generated inside is sent to the external analog feedback position control system through an 'k' bit digital to analog converter which decides the spatial resolution of the joint in focus.

Resolution for a Cylindrical Co-ordinated Robot

The work envelope of the cylindrical robot and the elemental sweep is as shown in the Fig. 2.9 (*a*). The horizontal precision is lowest at the outer most radius or reach and highest at the inner most reach.

The grid element shown in Fig. 2.9 (*b*) is not a square but a sector. But for a very small division it almost looks like a square.

Hence,

The radial precision (resolution) $= dr$

Minimum angular resolution $= rd\phi$

Worst angular resolution $= Rd\phi$.

Assuming grid element to be a square,

the horizontal resolution $= dh = \sqrt{(dr)^2 + (Rd\phi)^2}$ ...(2.7)

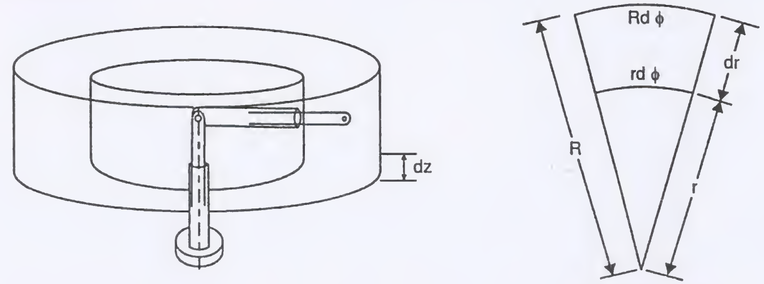
the vertical resolution $= dz$

Now,

the total control resolution is given by

$$dT = [(dr)^2 + (Rd\phi)^2 + (dz)^2]^{1/2} \qquad ...(2.8)$$

If a spherical robot is considered both the vertical and the horizontal precision are highest along the inside surface and decreases as the arm extends outward, and minimum at the outer most surface. If the robot with articulated joints is considered both the resolutions vary over the work space.



(*a*) (*b*) Total Resolution.

Fig. 2.9

Accuracy

"Accuracy is the measure of the robot's ability to orient and locate the tool tip at a desired target location in the prescribed work volume or envelop".

Accuracy is related to resolution because as the resolution value is less, the accuracy is more. So higher resolution gives better accuracy, the ability to achieve the prescribed target location. In a worst case the desired point may lie in between the two target points. The error in positioning is the other name to the inaccuracy given by the term,

$$\frac{\text{Control resolution}}{2} \leq \text{error}, \qquad ...(2.9)$$

where the mechanical components of inaccuracies are neglected as they are more complicated to define and quantify. Hence the precision related to the accuracy gives a picture of discrete grid nodes that can be visited by the wrist end or the tool tip within the work space. Hence, the best accuracy is half of the grid size as shown in the Fig. 2.10.

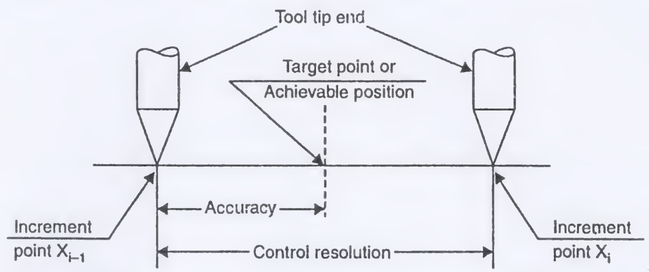
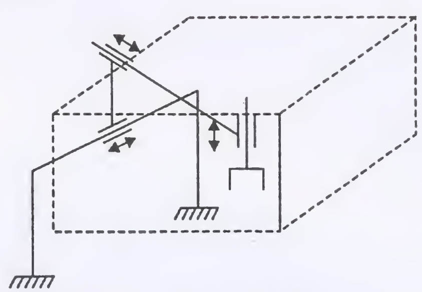


Fig. 2.10. Representation of Control Resolution and Accuracy.

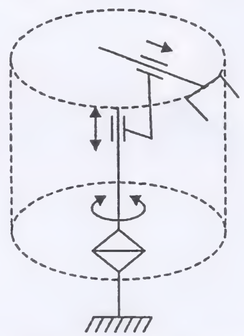
After a periodic operation set the robot may have to be calibrated to maintain the reasonable accuracy. The limit switch sensors of the robots are reset or zeroed during the periodic maintenance schedule. Further intelligent algorithms with real time solutions are needed to define and re-define the control strategies to compensate for the uncertainity in environment and position.

2.9.1 Types of Work Spaces

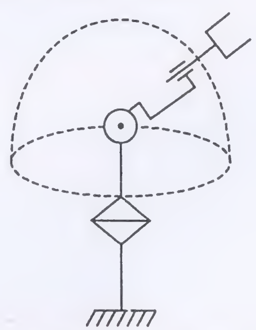
1. Cartisian Robot. This has three linear motions in x, y, and z direction. The work space covered is cuboidal type.



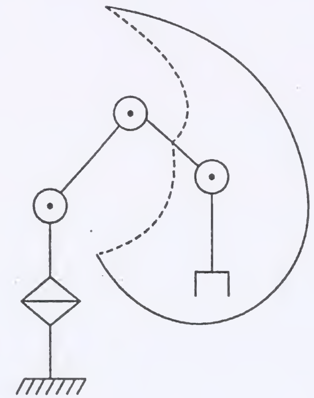
2. Cylindrical Robot. This has two linear motions in z and radial direction and a rotary motion about z-axis forming a cylindrical work envelope.



3. Polar Robot. This has one linear motion and two rotary joints moving about z and y axis. The work space generated is spherical in shape.

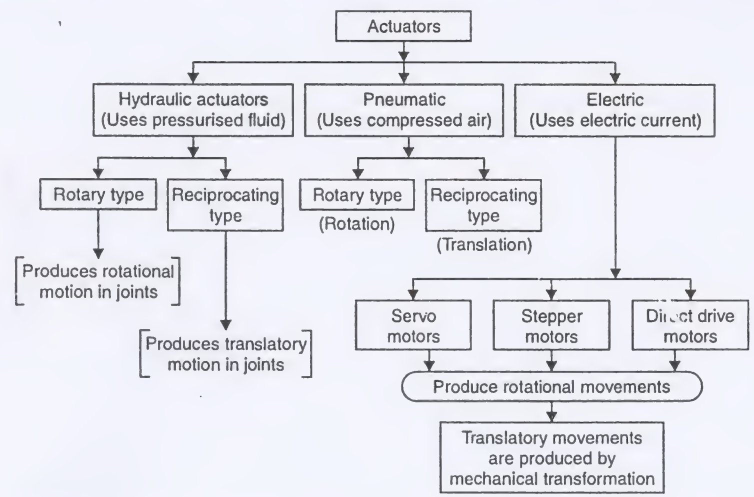


4. Combined Type (Revolute Robot). This has all the three joints revolute which produce rotary movements in x, y and z direction giving a irregular shaped work envelope.



2.10 ROBOT DRIVE SYSTEMS

The links of the robots move about the prescribed axis by receiving the power through, what are called the drive systems, also known as actuators. The movements produced may be translatory in nature or rotary about a joint. At the joints the actuators provide required force or torque for the movement of the links. The movements of all the links, combined together form the arm end or wrist motion. The source of power for the actuators can be through the compressed air, pressurised fluid or the electricity, based on which they are classified as follows.



2.11 HYDRAULIC ACTUATORS

The hydraulic actuators receive pressurised hydraulic oil with controlled direction and pressure through a system known as 'power packs'. The speed and volume flow rates are also controlled by the elements of the power pack. To produce linear motion the hydraulic cylinders

are used and hydraulic motors are used to produce rotational movements. A list of elements of a hydraulic power pack along with their function are given here.

Elements of Power pack and functions

	Elements	*Functions*
1.	Reservoir or tank	Stores and supply hydraulic oil to the system, in a closed circuit.
2.	Hydraulic pump	Receives oil from the reservoir and pressurises the oil in accordance with its capacity.
3.	Electric motor	Receives electric current from mains and provides rotational movement to the pump.
4.	Valves	Control the direction of flow, regulate the pressure and provide safety to the system.
5.	Hoses and pipes	Provide connection between the various elements transporting the high pressure oil.

• Hydraulic System

The hydraulic circuit shown in Fig. 2.11 (*a*) shows symbolically the hydraulic system of linear actuator in a simplest arrangement. On the forward stroke of the piston of the cylinder the high pressure relief valve is effective and in the return stroke the low pressure relief valve acts to regulate the system pressure. The load handling capacity is determined by the system pressure in the forward stroke. The directional control valve controls the direction of motion of the piston in case of the linear actuators.

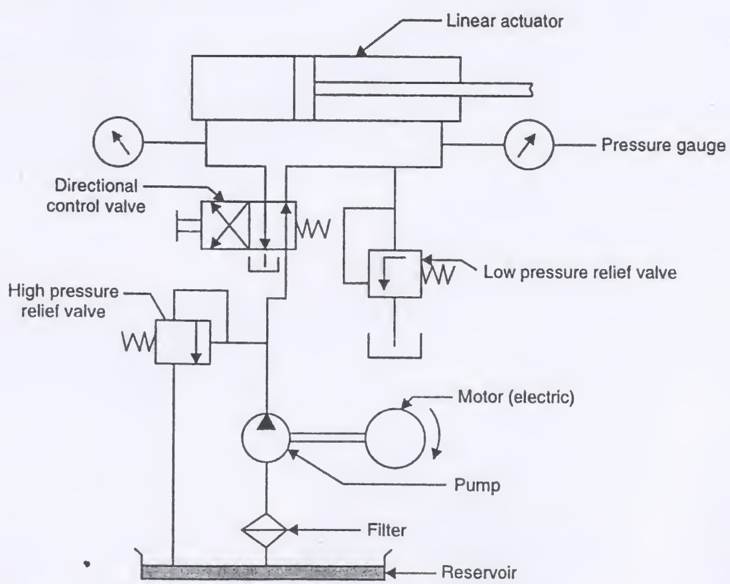


Fig. 2.11. (*a*) Typical Hydraulic Circuit.

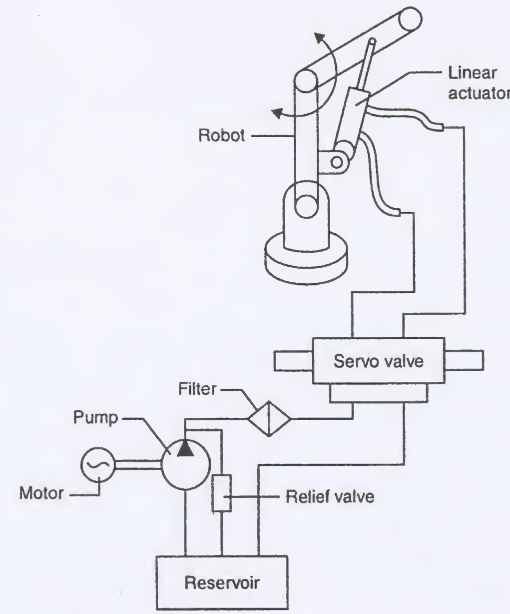


Fig. 2.11. (*b*) Hydraulic system for Robot.

• Hydraulic Motors

These are a type of power utilizing elements which convert hydraulic energy into mechanical rotational work useful in driving the links with revolute joints, in this context. One of the types of hydraulic motors, the vane motor is shown in Fig. 2.12, which is most commonly used type.

The motor consists of a circular rotor mounted ecentrically inside a circular stator ring. The rotor has got suitable slots for accommodating radially moving vanes. The rotational output of the motor depends on the eccentricity '*e*' of the rotor with respect to the stator.

The power output, $$P = \frac{D_r . e . V_r . p}{1000} \text{ kW} \quad ...(2.10)$$

where D_r = outermost diameter of the vane, *m*

e = ecentricity, *m*

V_r = linear speed of rotation = $\frac{2\pi NR}{60}$ m/sec

N = revolution per minute

$R = D_r/2$

p = pressure of the oil supplied to the motor.

And the torque developed, $$T = \frac{60{,}000 P}{2\pi N} \text{ kN-m} \quad ...(2.11)$$

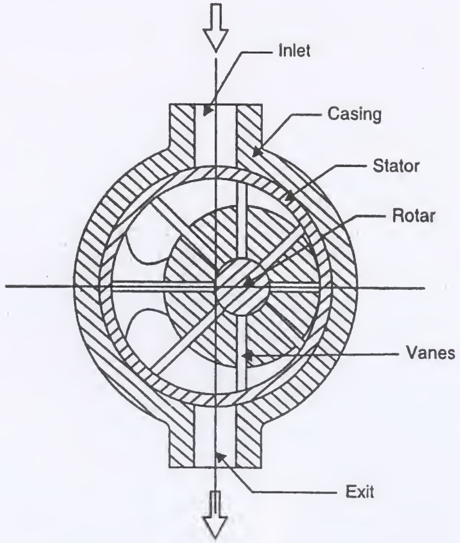


Fig. 2.12. Vane Motor.

• Linear Actuators

The actuators that provide linear reciprocating motion to the prismatic joints, by utilizing hydraulic power are known as linear actuators or cylinders, the constructional details are as shown in Fig. 2.13.

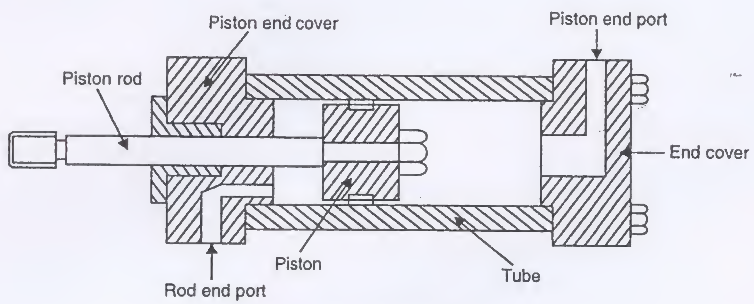


Fig. 2.13. Construction of Cylinder.

A cylinder essentially consists of a piston located in the tubular housing and a piston rod passing through one of the end covers. The ports provided in the end covers permit entry and return of the hydraulic oil.

If the hydraulic oil is supplied to the cylinder is at a pressure pN/m^2 with a volume flow rate of Q m^3/sec, the power delivered is given by

$$P = \frac{p \cdot Q}{1000} \text{ kW.} \quad \text{...(2.12)}$$

The force developed in the rod is given by

$$F = p \cdot \frac{\pi}{4} \cdot D_p^{\,2} \quad \text{...(2.13)}$$

where D_p = diameter of the piston in meter.

$$\text{The speed of motion,} \quad V_e = \frac{Q}{4} = \frac{4Q}{\pi D_p^{\,2}} \quad \text{...(2.14)}$$

This is the speed in extension of the rod.

$$\text{The speed of retraction, } V_r = \frac{4Q}{\pi(D_p^{\,2} - D_r^{\,2})} \quad \text{...(2.15)}$$

• Features of Hydraulic Actuators

- provide high power in small light components
- have flat load-speed or torque speed characteristics
- can operate safely and continuously under stall conditions
- provide stepless variation in speed
- have longer life and reliability due to the lubricating properties of the oil.
- can be easily built using readily available standard elements.
- have contaminant sensitive elements.
- the operation is noisy.
- higher inertia on the robot joints.
- power loss and unclean work area due to possibility of leak.
- less deflection due to low compliance of the elements.

• Applications of Hydraulic Actuators

1. Used to drive the spray coating robots.
2. Used in heavy part loading robots.
3. Useful in material handling robot system.
4. Used to drive the joints of assembly (heavy) robots.
5. Useful in producing translatory motion in cartesian robot.
6. Useful in robots operating in hazardous, sparking environments.
7. Useful in gripper mechanisms.

2.12 PNEUMATIC ACTUATORS

The principles of pneumatic actuators match with that of hydraulic actuator. The working fluid in case of this is the compressed air. The pressure of air used in this varies from 6–10 MPa. Because of low air pressure the components are light and the force/torque transmitted is also less, Pneumatic cylinders are used to actuate the linear joints and pneumatic motors are used to drive the revolute joints. The main problem with the pneumatic devices is that the working fluid (air) is compressible, hence the actuator drifts under loads.

The pneumatic actuators are characterised by the following features:

- Lowest power to weight ratio.
- Highly compliant system.
- Drift under load constantly.
- Low, inaccurate response due to low stiffness.
- Less leakage of air and not susceptible to sparks.
- Uses low pressure compressed air, hence less actuation force or torque.
- Useful in on-off applications like pick and place robots.
- Simple and low cost components.
- Reliable and easily available components.
- The exact positions of the actuators can be controlled by servocontrol valves by differential movements.

2.13 ELECTRIC DRIVES

• Principle

A rotational movement is produced in a rotor when an electric current flows through the windings of the armature setting up a magnetic field opposing the field set up by the magnets.

• The Main Components

Rotor, stator, brush and commutator assembly. The rotor has got windings of armature and the stator has got the magnet. The brush and the commutator assemblies switch the current to the armature maintaining an opposed field in the magnets.

• Performance

The torque on the rotor of the electric motor is given as

$$T_m = K_m \cdot I_a, \qquad \text{...(2.16)}$$

where K_m = motor torque constant

I_a = armature current.

$$= \frac{V_{in} - e_b}{R_a}, \qquad \text{...(2.17)}$$

where V_{in} = input voltage to the motor

e_b = back e.m.f.

R_a = armature resistance,

But e_b is the back e.m.f., the opposing voltage produced in the winding due to the rotation of the rotor. e_b is given as

$$e_b = K_b \cdot \omega \qquad \text{...(2.18)}$$

where K_b = voltage constant, ω is the angular velocity.

• Selection

The selection of the electric drive is based on the torque rating and the current rating of the motor. The torque rating of a electric motor is derived from the power rating of the motor. If P_m is the power rating the torque rating

$$T_r = \frac{60\, P_m}{2\pi\, N_m}, \quad \text{where,} \quad N_m = \text{speed specification of the motor.}$$

• Types

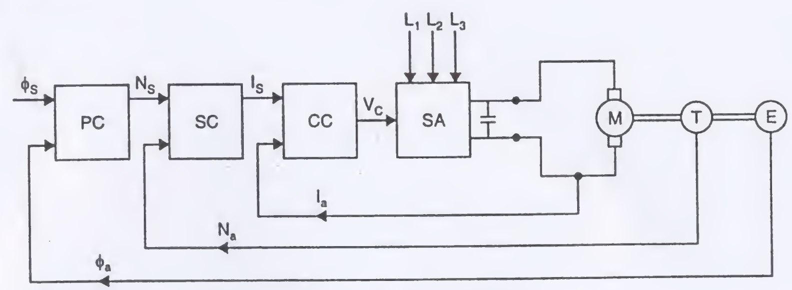
The most commonly used electric drives in robotics are

1. DC Servo motor
2. AC Servo motor
3. Stepper motor.

• Features

DC Servo Motors	*AC Servo Motors*	*Stepper Motors*
• Higher power to weight ratio.	• Rotor is a permanent magnet and stator is housing the winding.	• Moves in known angle of rotation.
• High acceleration.	• No commutators and brushes.	• Position feed back is not necessary.
• Uniform torque.	• Switch is due to AC but not by commutation.	• Rotation of the shaft by rotation of the magnetic field.
• Good response for better control.	• Fixed nominal speed.	• Needs microprocessor circuit to start.
• Reliable, sturdy and powerful.	• Favourable heat dissipation	• Used in table top robots.
• Produces sparks in operation, not suitable for certain environments.	• More powerful.	• Finds less use in industrial robots.
	• Reversibility of rotation possible.	• Extensive use is robotic devices.

• Servo Motors



PC = Position controller
SC = Speed controller
CC = Current controller
SA = Servo amplifier
M = DC/AC motor
T = Tacho-generator
E = Encoder.

N_a = actual speed
ϕ_a = actual position
I_a = actual current
L_1, L_2, L_3 = 3-phase supply.
ϕ_s, N_s, I_s, = desired position, speed and current
V_c = control voltage.

Fig. 2.14 (*a*)

The desired position (ϕ_s) is compared with the actual position (ϕ_a) feedback from the encoder (E). This gives the desired speed (N_s), which is compared with the actual speed (N_a) obtained as feedback from the tacho-generator (T). This gives the desired current (I_s) which is adjusted by the inner loop giving a feedback of actual current (I_a). A control signal (V_c) is generated which alongwith supply voltage V_d from a 3-phase system is given to the motor as input. In a servo-motor the position and speed of motor is controlled by the feedback control. The block diagram of servo motor is shown in Fig. 2.14(*a*).

2.14 STEPPER MOTORS

The stepper motors are unique type of motors that produce rotational movement in the form of finite angular steps. The intermittent electrical pulses make the stepper motor shaft to rotate in steps.

The Fig. 2.14 (*b*) shows the schematic arrangement of the stepper motor principle. The stator in this case is made up of four-electromagnetic poles. The rotor is a permanent magnet with two poles N and S. When the excitation of pole 2 (P2) is changed to P3 pole the magnetic north of the rotates by 90° clockwise. By continuous change in excitation in the order P2-P3-P4-P1-P2 the clockwise rotation is produced in the shaft of the rotor, which results in continuous movement.

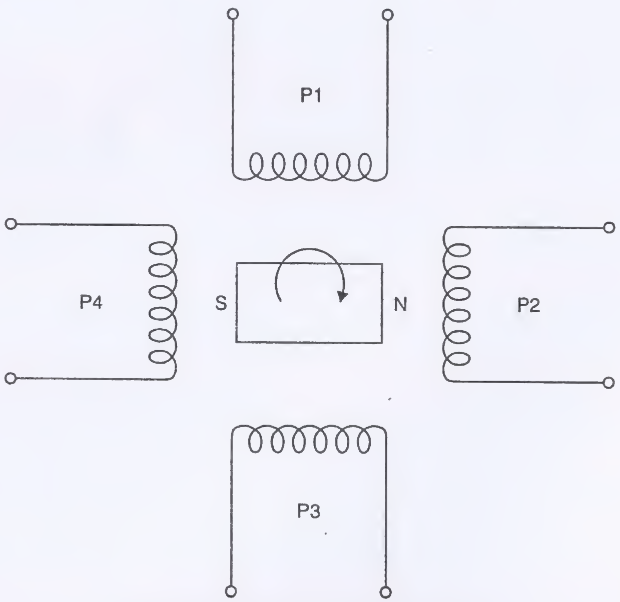


Fig. 2.14. (*b*) Schematic of Stepper Motor.

• Construction of Stepper Motor

Multiple pole stepper Motor :

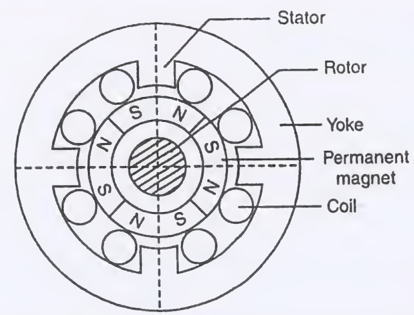


Fig. 2.14. (*c*) Permanent magnet stepper motor.

The stator has a winding made of concentrated coils on distinct poles. The rotor is permanent magnet cylinder.

Single-phase stepper motor:

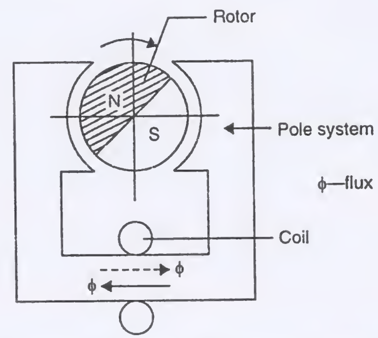


Fig. 2.14. (*d*) Single phase stepper motor.

Single phase stepper motor with two poles is shown in Fig. 2.14(*d*). By reversal of the current in the coil the polarity changes continuously by change in flow of flux in the poles. The rotor which is the permanent magnet makes rotation.

Resolution of Stepper Motor

Resolution is determined by the number of stator poles in the stepper motor.

Step angle, $$A_s = \frac{360°}{n_s}, \quad ...(2.19)$$

where n_s is number of poles

or $$n_s = \frac{360°}{A_s}$$

The resolution is given by the inverse of the number of steps or the poles,

$$R_s = \frac{A_s}{360°} \quad ...(2.20)$$

Pulse:

Single pulse of electrical signal is necessary for the rotor to rotate by one step. For one rotation number of pulse,

$$n_p = n_s = \frac{360°}{A_s} \qquad \text{...(2.21)}$$

The pulse count for N_R revolutions of the rotor

$$n = N_R \cdot n_p$$

From (2.21), $$n = \left(\frac{360°}{A_s}\right) \cdot N_R$$

or $$n = \frac{N_R}{R_s} \quad \text{as} \quad R_s = \frac{A_s}{360°} \text{ from (2.20)}$$

Hence the pulse count is the ratio of number of revolutions and the resolution of the stepper motor.

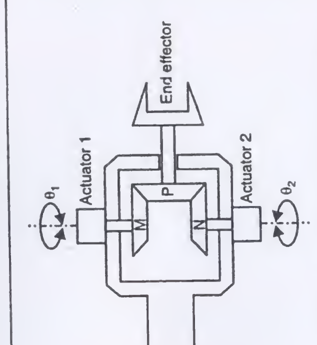
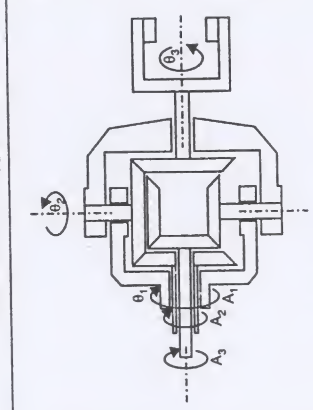
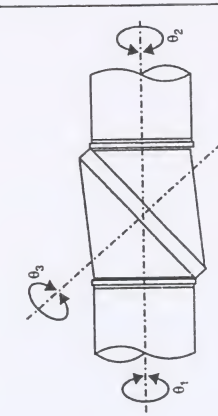
If the rotor is rotating with a speed of N_m revolutions per minute,

The pulse rate, $$n_r = \frac{N_m}{R_s} = \frac{\text{number of revolution / min}}{\text{Resolution}}$$

2.15 COMPARISION OF CHARACTERISTICS OF ROBOT DRIVE SYSTEMS

Sl. No.	*Comparing Features*	*Hydraulic Drive*	*Electric Drive*	*Pneumatic Drive*
1.	Power to weight-ratio—	• Highest	• Moderate	• Lowest
2.	Payload carried by the robot—	• Heavy	• Medium	• Low
3.	Controlling devices—	• Needs a hydraulic power pack	• Control system is needed	• Pneumatic power control devices needed
4.	Size and stiffness—	• Very high.	• Low stiffness	• Very low
5.	Compliance of the system—	• Low	• Better	• Good
6.	Leakage and cleanliness—	• Worst	• Nil	• Better
7.	Reliability of the components—	• Low	• High	• Higher
8.	Accuracy and response—	• Good	• Higher	• Bad
9.	Need for maintainance—	• Needed more	• Low	• Less
10.	Pressure, Torque and inertia on the actuator—	• High	• Medium to high	• Low to medium
11.	Range of operational speeds—	• Wide	• Comparatively less	• Very little
12.	Striking or generation of spark—	• Not there	• Possible	• No sparks
13.	Path generation application—	• Continuous path	• Both continuous pick and place	• Only in pick and place types

2.16 WRISTS AND MOTIONS

Orthogonal Axis Wrists		*Non-Orthogonal Axis Wrists*
Pitch-Roll Wrist	*Pitch-Roll-Yaw type Wrist*	
 Fig. 2.15. Pitch-Roll Gripper Wrist. *Features and Applications* • Uses set of three (M, N, P) bevel gears • Configuration resembles a "universal joint" • The shafts of M and N are driven beg two separate stepper motors • Pure "roll motion" is produced by when M and N are driven in opposite direction with same speed. • Pure "pitch motion" is exhibited by the motion of M and N in the same direction with same speed. • Applied to hold the tool (electrode) in a welding robot.	 Fig. 2.16. Non-orthogonal Wrist. *Features and Applications* • Uses set of five bevel gears • Actuators are remotely located. • By different combinations of rotation and speed of the actuators the wrist attains pitch-roll-yaw motions. • It has got three consecutive intersecting axes, which are orthogonal. • Applied to hold the gripper in assembly and handling operations. • This has three intersecting but non-orthogonal axes.	 Fig. 2.17. Pitch-Roll-yaw Wrist. *Features and Applications* • This is also called "Three Roll Wrist". • All three joints of the wrist rotate continuously driven by actuators. • This has got some unattainable orientations. • Design of such wrist is complicated. • Driven by two actuators, to produce three rolls of movements. • Used in pick and place, assembly robot. ex. Cincinatti Milacron.

2.17 DESIGNS OF GRIPPER FINGERS

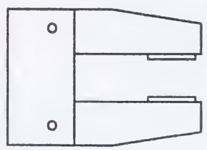
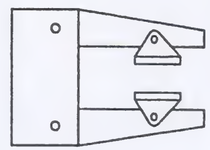
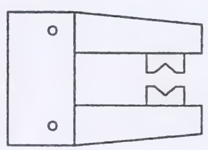
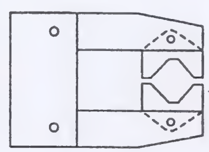
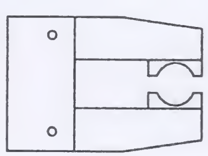
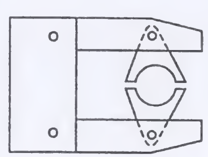
Pivot Type / Pad Shapes	Mechanical Types: Fixed Pivot	Mechanical Types: Hinged Pivot	Non-Mechanical Types
1. Flat type			1. Vacuum Cup Type: • The objects have to be flat smooth and clean. • Cups are made of elastic materials round in shape • Vacuum is created between cup and the object (glass) 2. Adhesive Gripper: • Used to handle fabrics and light weight materials. • Gripping ability diminishes with successive operations. • Adhesive ribbon is alternative. 3. Magnetic Gripper: • Fast pickup time. • Variety of part sizes can be handled. • Metal parts with holes can be easily handled. • Requires only one surface for gripping. 4. Expandable Bladder Type: • In flatable bladder expands to grab the object. • Bladder is made of elastic material, which applies uniform grasping pressure. • Used to handle fragile parts.
2. V-block type			
3. Circular arc type			

Fig. 2.18

2.18 GRIPPER (END EFFECTOR) MECHANISMS

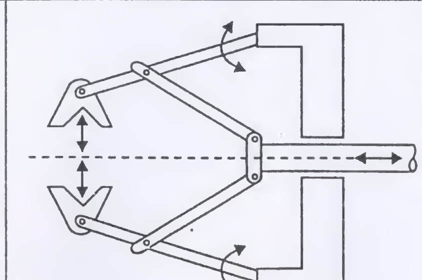
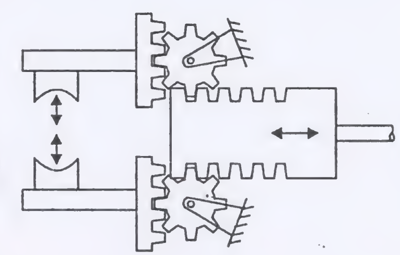
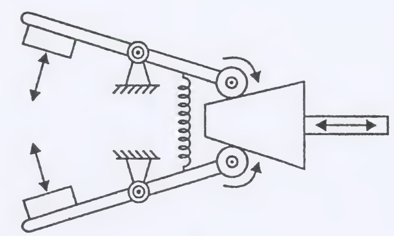
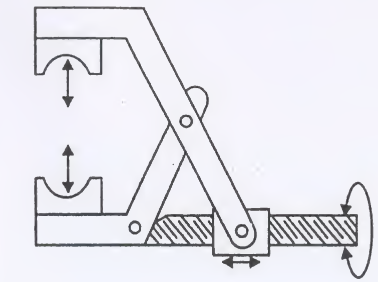
Mechanism			Description
1. Linkage Actuation		(i)	• Many design possibility using linkages. • Higher gripping force at less actuating force due to leverage. • Opening width and the speed of gripping depends on the linkage configuration.
2. Rack-pinion Actuation		(ii)	• A rack actuated by a hydraulic/pneumatic cylinder operates two pivoted pinions. • Pinions oscillate to move the racks (two) • The fingers attached to the racks (two) open and close to grasp and release the object.
3. Cam Actuation		(iii)	• A cam attached to a cylinder moves to and fro. • Spring-loaded followers moves on the cam profile. • The finger pads attached to the pivoted liver connected to the followers open and close to release and grab the object.
4. Screw Actuation		(iv)	• A threaded block engages with a screw moves forward and backward, the fingers attached to levers connected to the threaded block by a hinged joint, open and close providing releasing and gripping action. • The screw is rotated by a motor through reduction box.

Fig. 2.19 (a)

2.18a PNEUMATIC GRIPPERS

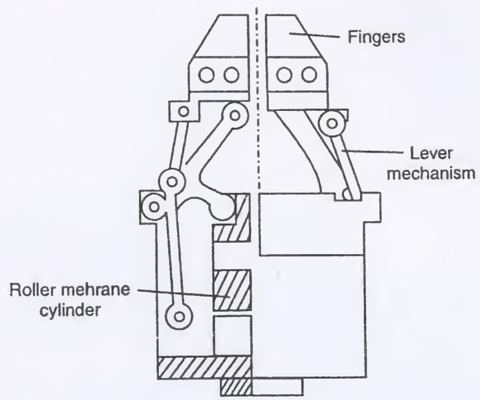


Fig. 2.19. (*b*) Pneumatic Gripper.

This pneumatic gripper is equipped with roller membrane cylinder with a rolling motion replacing conventional piston cylinder. The motion is transmitted to gripper fingers by means of lever mechanism. The grippers are actuated by switching valves in the circuit. The finger stroke is limited by end stops or the workpiece to be gripped. The gripping force is determined by the pressure of air applied and the leverage.

2.19 FORCE ANALYSIS OF GRIPPER MECHANISM

A gripper mechanism consisting of fingers, linkages frame and a pneumatic cylinder is shown in Fig. 2.20. Air pressure supplied to the cylinder aids in actuating the fingers to grab an object with a gripper force P_g.

If the mass of the object is 'm' and 'g' is gravity acceleration.

The force due to mass $= m.g. = W$, newtons. ...(2.22)

The friction between the finger pads is responsible for the gripper to hold the object exerting the force W.

The friction force is given by,

$$f = \mu\, NP_g, \qquad ...(2.23\,a)$$

where μ = coefficient of friction,

N = the number of fingers.

Due to the uncertaining of circumstances the capacity of the fingers had to be increased due to incorporate a safety by a factor of safety, n.

i.e., F_d = design force $= n.W$ ...(2.23 *b*)

Equating equations (2.23 *a*) and (2.23 *b*)

$$n.W = \mu\, NP_g$$

and
$$P_g = \frac{n.W}{\mu N} = \frac{n.m.g}{\mu N} \qquad ...(2.24)$$

If the gripper is accelerating or decelerating by 'a'

$$P_g = \frac{n.m}{\mu N}(g \pm a) \qquad ...(2.25)$$

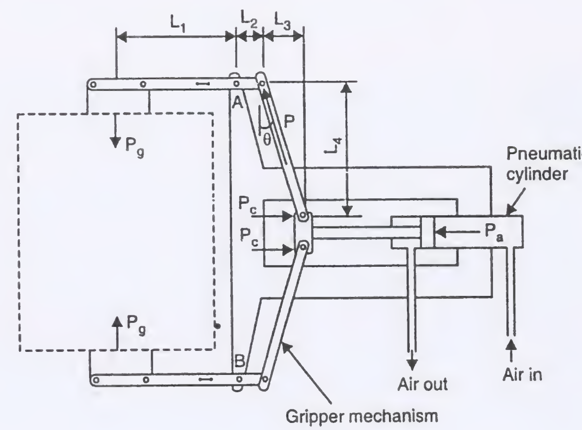


Fig. 2.20. Force Analysis of Gripper Mechanism.

'a' takes the positive sign when accelerating down and takes negative sign when accelerating up.

The expression (2.25) can also be written as

$$\boxed{P_g = \frac{n.(mg)}{\mu N}(1 \pm K_f)} \qquad ...(2.26)$$

The factor $K_f = \dfrac{a}{g}$ and weight of the object, $W = (mg)$

Table 2.3

Accelerating motion	*Factor* $(1 \pm K_f)$
• Accelerating down	3
• Accelerating up	1
• Accelerating in horizontal direction	2

• Linkage Analysis

Referring to the Fig. 2.20 and taking moment about the point A,

$$P_g \,.\, L_1 - (P \cos\theta) \,.\, L_2 = 0$$

$$P = \frac{P_g \,.\, L_1}{(L_2 \,.\, \cos\theta)} \qquad ...(2.27)$$

where $\theta = \tan^{-1}\left(\frac{L_3}{L_4}\right)$ and P = linkage force in Newton Resolving the force, P

$$P_c = P \,.\, \sin\theta$$

Substituting equation (2.27)

$$P_c = \frac{P_g \,.\, L_1}{L_2} \,.\, \tan\theta.$$

But the force to be exerted by the piston on linkages

$$P_a = 2P_c$$

Using equations (2.26) and (2.27)

$$\boxed{P_a = \frac{2n\,W}{\mu\,N}\left(\frac{L_1}{L_2}\right)(1 \pm K_f).\ \tan\theta} \qquad ...(2.28)$$

Substituting $\tan\theta = \frac{L_3}{L_4}$

$$P_a = \frac{2nW}{\mu\,N}\left(\frac{L_1 \,.\, L_3}{L_2\,L_4}\right)(1 \pm K_f) \qquad ...(2.29)$$

Power requirements:

If the diameter of the piston of the actuator is d_p

Area of the piston, $A_p = \frac{\pi}{4} \cdot d_p^{\,2}$ in m^2

The pressure of air needed $= \frac{P_a}{A_p} = p$

$$p = \frac{4}{\pi} \cdot \frac{P_a}{d_p^{\,2}}$$

The power required to produce this pressure, $p(N/m^2)$

$$P_R = \frac{p \,.\, Q}{1000}\ kW \qquad ...(2.30)$$

where Q is the discharge (volume flow rate) of air in m^3/sec.

2.20 GRIPPER DESIGN CONSIDERATION

Part Specification	*Performance Specification*	*Source Specification*	*Position Specification*	*Environmental Specification*	*Material Specification*
• Weight of the part • Size and shape of the component • Tolerance on the part size. • Change of shape and size during processing. • Surface finish of the part. • Care for delicacy of the part to be handled.	• Co-efficient of friction between part and object. • Speed and acceleration during motion. • Accuracy and repeatability of the robot. • Interchangeability of fingers. • Memory capacity of the controller. • Control resolution of the robot. • Lead time and cycle time.	• Pneumatic: air pressure and discharge, and cylinder size. • Electrical: The power rating and specification of actuator. • Hydraulic: The oil pressure, volume flow rate and the power pack specification. • Mechanical: Linkage design and transmission design specification.	• Holding methods. • Physical difficulty • Length of fingers • The object and tool orientation. • Product design changes. • Spare parts specifications.	• Heat and temperature of object and atmosphere. • Humidity and moisture. • Dirty and safety. • Hazardness of chemicals used.	• Strength and rigidity. • Durability and fatigue factors. • Friction properties. • Factor of safety. • Design standards. • Compatibility with the environment.

2.21 SELECTION CONSIDERATION OF GRIPPER (END-EFFECTOR)

Actuation Selection	*Drive Selection*	*Protection Selection*	*Process Selection*
• Mechanical or friction gripping methods. • Pad shape selection. • Vacuum actuation • Magnetic grasping. • Adhesive gripping. • Expandable bladder type actuation.	• Pneumatic drive systems. • Hydraulic drive System for heavy duty operation. • Electrical drive for light duty application. • Speed reduction of the mechanical transmission.	• Heat shield for sensors and actuators. • Forced cooling by air or water cooling to take away the heat. • Selection of heat resistance materials for fingers and components of gripper. • Shield from hazardous chemicals.	• Accurate processing methods for fingers. • Leak prevention for pneumatic/hydraulic actuators. • Interchangeability standards for the fingers. • Shape compatible processing methods for fingers. • Ease of assembly of fingers and linkages.

PROBLEMS

Example 2.1. *A cylindrical co-ordinated robot has a vertical reach of 500 mm and a stroke of 320 mm. What is the minimum height of the work table to be to able the robot to reach the object kept on the table?*

Sol. Data:

V = maximum vertical reach = 500 mm.

L = length of the stroke = 320 mm.

H = minimum height of the table = ?

$$H = (V - L) = (500 - 320) = 180 \text{ mm.}$$

Ans. Minimum height of the table = 180 mm.

Example 2.2. *A cartesian robot has a horizontal reach of 450 mm and a horizontal stroke of 250 mm. What is the maximum limit within which the object placed is not reachable?*

Sol. Data:

H = maximum horizontal reach = 450 mm.

S = stroke length = 250 mm.

L = non-reachable maximum limit = ?

$$L = (H - S) = (450 - 250) = 200.$$

Ans. Non-reachable maximum limit = 200 mm.

Example 2.3. *One of the links of a robot has a telescoping arm with a stroke of 512 mm. The control memory of the robot has 8-bit storage capacity for this axis. Determine the control resolution for the same.*

Sol. Data:

S = stroke length = 512 mm.

K = storage capacity = 8 bit memory.

From the equation (2.2),

Total control resolution,

$$R_t = \frac{S}{(2)^k} = \frac{512}{(2)^8} = 2 \text{ mm.}$$

Ans. Total control resolution = 2 mm.

Example 2.4. *A cartesian robot has a slide with a total range of 1.2 m and it is desired that it will have a control resolution of 0.46 cm on this axis. Determine the bit storage capacity which the control memory must possess to accomodate this level of precision.*

(VTU, Jan./Feb. 2003)

Sol. Data:

Total range of linear movement, = S = 1.2 m (1200 mm)

Control resolution, R_t = 0.46 cm (4.6 mm)

From the equation (2.2).

$$R_t = \frac{S}{2^k}$$

$$2^k = \frac{S}{R_t}$$

$$k(\log_e 2) = \log_e \left(\frac{S}{R_t}\right)$$

$$k = \log_e \left(\frac{S}{R_t}\right) \Big/ \log_e (2) \quad \ldots(i)$$

$$= \frac{\log_e \left(\frac{1200}{4.6}\right)}{\log_e (2)} = 8.027.$$

Ans. The number of bits of storage capacity = 8.

Example 2.5. *A large cartesian co-ordinate robot has one orthogonal slide with a total stroke of 1024 mm. This axis has the maximum control resolution of 0.25 mm. Determine the number of bit storage capacity which the robot control memory must possess to provide this level of precision.*

Sol. Data:

Total stroke, S = 1024 mm

Control resolution, R_t = 0.25 mm

The number of bits of storage capacity, from relation (*a*) of example 2.4.

$$k = \frac{\log_e [S/R_t]}{\log_e [2]}$$

$$= \frac{\log_e [1024/0.25]}{\log_e (2)} = 12.$$

Ans. The number of bit storage capacity = 12.

Example 2.6. *The telescopic arm of an industrial robot obtains total range of rotation of 120°. The robot has a 12 bit storage capacity for the axis. The arm fully extends to 1500 mm and fully retracts to 750 mm from the pivot point. Determine the robots control resolution (i) for the axis in degrees of rotation and (ii) on linear scale in fully extended and retracted position.*

(VTU, May/June 2004)

Sol. Data:

Range of rotation, $\phi = 120°$

Number of bit storage capacity, $k = 12$

Maximum reach of extension, $H_{max} = 1500$ mm

Minimum reach of retraction, $H_{min} = 750$ mm.

Control resolution of rotation, $A_c = ?$

Control resolution of linear scale, $R_t = ?$

(*i*) Using the equation (2.5),

The control resolution of rotation,

$$A_c = \frac{\phi}{2^k} = \frac{120}{(2)^{12}} = 0.0293°$$

(*ii*) Stroke $S = (H_{max} - H_{min})$

$= (1500 - 750) = 750$ mm.

From equation (2.2),

$$R_t = \frac{S}{2^k} = \frac{750}{(2)^{12}} = 0.183 \text{ mm.}$$

Ans. (*i*) Control resolution for rotation = 0.0293°

(*ii*) Control resolution for translation = 0.183 mm.

Example 2.7. *The mechanism connecting the wrist assembly is a twisting joint which can be rotated through 8 full revolutions from the start to end position. It is desired to have control resolution of rotation of ± 0.35° at the least. What is number of bit storage capacity to achieve this resolution?*

Sol. Data:

The degrees of rotation = 360° × no. of revolution

i.e., $\phi = 8 \times 360° = 2880°$

Total resolution $= \pm 0.35°$

So, control resolution $= A_c = 0.70°$

Using the equation (2.5),

$$A_c = \frac{\phi}{2^k} \quad \text{or} \quad 2^k = \frac{\phi}{A_c}$$

$$k \,.\, \log_e(2) = \log_e [\phi/A_c]$$

$$k = \frac{\log_e [\phi/A_c]}{\log_e (2)} = \frac{\log_e (2880/0.7)}{\log_e (2)} = 12.006.$$

Ans. The number of bit storage capacity = 12.

Example 2.8. *An incremental shaft encoder with 2 emitter detector pairs and 12 slots around the circumference is used to monitor the angular position of a high speed motor shaft. The precision of the load shaft is measured and found to be 0.05 degree per count. What is the gear ratio between high speed shaft and the load shaft?*

Sol. Using the equation (2.4),

$$A_c = \frac{2\pi}{nZ \,.\, 2^k} = \frac{360}{nZ \,.\, 2^k},$$

where n = number of slots = 12,

k = number of emitter detector pairs, = 2,

Z = speed reduction ratio = ?,

A_c = The angular control resolution, = 0.05° (precision).

Hence
$$Z = \frac{360}{n \,.\, A_c \,.\, 2^k}$$

$$= \frac{360}{(12)(0.05)\,.\,(2)^2} = 150.$$

Ans. The gear reduction ratio between high speed shaft and the load shaft = 150 : 1.

Example 2.9. *A cylindrical robot has a prismatic joint with a range of travel of 800 mm. The control memory for this joint has 10 bit capacity. It has been recorded that the associated mechanical inaccuracies with the said arm show a random distribution of random variable of the robot position about the mean position of the taught point gives a standard deviation of 0.1 mm. The standard deviation is equal in all direction. Determine the following:*

(*a*) *The control resolution for the axis.*

(*b*) *The spatial resolution for the prismatic joint.*

(*c*) *The accuracy defined.*

(*d*) *The repeatability of the robot link.*

Sol. Data:

The stroke, $S = 800$ mm

The bit storage capacity, $k = 10$

The standard deviation of the mechanical inaccuracies,

$\sigma = 0.1$ mm.

(*a*) Control resolution,

Using the equation (2.2),

$$R_t = \frac{S}{2^k} = \frac{800}{(2)^{10}} = 0.78125 \text{ mm.}$$

(*b*) The spatial resolution,

From the equation (2.6),

$$R_s = R_t + 6\sigma$$

$$= 0.78125 + 6(0.1) = 1.38125 \text{ mm.}$$

(*c*) The accuracy

Using the relation (2.9),

$$\text{error (accuracy)} \geq \frac{R_t}{2}$$

$$\text{accuracy} \geq \frac{0.78125}{2} = 0.3906 \text{ mm.}$$

(*d*) The repeatability $R_p = \pm 3\sigma$

$= \pm 3(0.1) = \pm 0.3$ mm.

Ans. (*a*) Control resolution = 0.78125 mm,

(*b*) Spatial resolution = 1.38125 mm,

(*c*) The accuracy ≥ 0.3906 mm,

(*d*) The repeatability = ± 0.3 mm.

Example 2.10. *The base joint of the cylindrical robot is driven by a 12-bit memory converter and has a swing range of 360°. The radial axis is driven by an 8-bit memory converter, and has a horizontal reach of 300 mm and a stroke of 200 nm. The vertical motion has a drive of 10 bit memory converter with a vertical reach of 480 mm and a stroke of 360 mm. Compute the following.*

(*a*) *Volume of the work envelope.* (*b*) *Radial resolution.*

(*c*) *Vertical resolution.* (*d*) *Angular resolution.*

(*e*) *Horizontal resolution.* (*f*) *The total resolution.*

Sol. (*a*) Volume of the work envelope,

$$V = \pi(r_1^2 - r_2^2)\, h$$

where $r_1 = 300$ mm, $r_2 = (r_1 - s) = (300 - 200) = 100$ mm.

$$h = (480 - 360) = 120 \text{ mm}.$$

Hence, $V = \pi(300^2 - 100^2)(120) = 30.16 \times 10^6 \text{ mm}^3$.

(*b*) Radial resolution,

$$dr = \frac{(r_1 - r_2)}{2^{k_r}}, \quad \text{where} \quad k_r = 8$$

$$= \frac{(300 - 100)}{2^8} = 0.78125 \text{ mm}.$$

(*c*) Vertical resolution,

$$dz = \frac{(Z_1 - Z_2)}{2^{k_z}}$$

where $k_z = 10$, $Z_1 = 480$ mm., $Z_2 = (480 - 360) = 120$ mm.

$$= \frac{(480 - 120)}{2^{10}} = 0.3516 \text{ mm}.$$

(*d*) Angular resolution

$$d\phi = \frac{\phi}{2^{k_\phi}} \quad \text{where} \quad K_\phi = 12, \quad \phi = 360°$$

$$= \frac{360}{2^{12}} = 0.088 \text{ deg } (1.534 \times 10^{-3} \text{ radian})$$

Minimum angular resolution $= r_2\, d\phi = 100 \times 1.534 \times 10^{-3}$

$= 0.1534$ mm.

Maximum angular resolution, $= r_1\, d\phi = 300 \times 1.534 \times 10^{-3}$

$= 0.46$ mm.

(*e*) Horizontal resolution,

$$dh = [(dr)^2 + (Rd\phi)^2]^{1/2}$$

$$= [(0.78125)^2 + (0.46)^2]^{1/2} = 0.907 \text{ mm}.$$

(*f*) Total resolution,

Using relation (2.8), $$dT = \sqrt{(dr)^2 + (Rd\phi)^2 + (dz)^2}$$

$$= \sqrt{(0.78125)^2 + (0.46)^2 + (0.3516)^2} = 0.973 \text{ mm}.$$

Ans. (*a*) Work volume = 30.16×10^6 mm^3

(*b*) Radial resolution = 0.78125 mm

(*c*) Vertical resolution = 0.3516 mm.

(*d*) Angular resolution, minimum = 0.1534 mm

maximum = 0.46 mm.

(*e*) Horizontal resolution = 0.907 mm

(*f*) Total resolution = 0.973 mm.

Example 2.11. *A double acting, single-ended piston hydraulic cylinder is used to actuate one of the linear arm joints of a cartesian robot. The diameter of the piston is 10 cm and the diameter of the rod is 5 cm. A pump supplies hydraulic oil at a rate of 36 cm³/sec with a pressure of 40 N/cm². Determine :*

(*a*) *The force that can be applied by the piston in the forward and the reverse strokes.*

(*b*) *The maximum velocity with which rod can operate in forward and reverse directions.*

Sol. Data:

The diameter of the piston	$= D_p = 10$ cm (0.1 m)
The diameter of the rod,	$D_r = 5$ cm (0.05 m)
Discharge rate to the cylinder,	$Q = 36$ cm³/sec (3.6×10^{-5} m³/s)
The supply pressure of oil,	$p = 40$ N/cm² = (4×10^5 N/m²)

(*a*) The force exerted on the piston

* in the forward stroke $= p \,.\, A_p$

$$= \frac{\pi D_p^2 \,.\, p}{4} = \frac{\pi (0.1)^2 \times 4 \times 10^5}{4}$$

$$F_f = 3142 \text{ N}.$$

* in the reverse stroke, $F_r = p \,.\, (A_p - A_r)$

$$= p \,.\, \frac{\pi}{4}(D_p^2 - D_r^2)$$

$$F_r = (4 \times 10^5)\frac{\pi}{4}(0.1^2 - 0.05^2) = 2356.2 \text{ N}.$$

(*b*) The maximum velocity of the rod

* In the forward stroke,

$$(V_{max})_f = \frac{Q}{A_p} = \frac{4Q}{\pi D_p^2} = \frac{4 \times 3.6 \times 10^{-5}}{\pi (0.1)^2}$$

$$= 4.58 \times 10^{-3} \text{ m/sec} \qquad (4.58 \text{ mm/sec})$$

*In the reverse stroke,

$$(V_{max})_r = \frac{Q}{(A_p - A_r)} = \frac{4Q}{\pi(D_p^2 - D_r^2)}$$

$$= \frac{4 \times 3.6 \times 10^{-5}}{\pi(0.1^2 - 0.05^2)}$$

$$= 6.11 \times 10^{-3} \text{ m/sec } (6.11 \text{ mm/sec}).$$

Ans. (*a*) Force exerted

in the forward stroke = 3142 N.

in the reverse stroke = 2356.2 N.

(*b*) Velocity of rod

in the forward stroke = 4.58 mm/sec

in the reverse stroke = 6.11 mm/sec.

Example 2.12. *A hydraulic rotary van actuator is used to drive the revolute joint of a cylindrical robot with the power source delivering 27 cm³/sec of oil at a pressure of 705 N/cm². The outer and the inner vane radius are 10 cm and 5 cm respectively. The thickness of the vane is 1 cm. Determine :*

(*a*) *The angular velocity of the motor.*

(*b*) *The torque developed in the motor shaft.*

Sol. Data :

Outer radius of vane, R = 10 cm

Inner radius of vane, r = 5 cm

Discharge rate, Q = 27 cm³/sec.

Supply pressure, p = 705 N/cm²

Thickness of the vane, t = 1 cm

(*a*) Angular velocity,

$$\omega = \frac{2\theta}{(R^2 - r^2).t} = \frac{2 \times 27}{(10^2 - 5^2) \times 1.0} = 0\,.\,72 \text{ rad/sec.}$$

(*b*) The torque developed,

$$T = \frac{1}{2} \cdot \frac{p\,.\,t(R^2 - r^2)}{100}$$

$$= \frac{1}{2}\,.\,(705) \times 1.0 \times (10^2 - 5^2) = 264.4 \text{ N.m.}$$

Ans. (*a*) The angular velocity, ω = 0.72

(*b*) The torque developed, T = 264.4 N.m.

Example 2.13. *A DC servomotor used to drive a robot joint has a torque constant, 1.25 N-m/A and a voltage constant of 12 × 10⁻³ V/rpm. The armature resistance of 2.5 ohms. A voltage of 25 V is applied at a point of time of robot cycle when the joint is stationary.*

Determine,

(*a*) *the torque of the motor immediately after the voltage is applied.*

(*b*) *the back e.m.f. and the respective torque corresponding to rotational speeds of 250 rmp. and 500 rpm.*

Sol. Data :

Torque constant, K_m = 1.25 N-m/A.

Voltage constant, $K_b = 12 \times 10^{-3}$ V/rpm.

Armature resistance, R_a = 2.5 ohm.

Starting voltage applied, V_{in} = 25 V.

(*a*) Torque at the start of rotation

Using the equations (2.16), (2.17) and (2.18)

$$T_m = K_m I_a$$

$$= \frac{K_m(V_{in} - e_b)}{R_a} = \frac{K_m(V_{in} - K_b\,.\,\omega)}{R_a}$$

at the start ω = 0

So $$T_m = \frac{K_m V_{in}}{R_a} = \frac{1.25(25)}{2.5} = 12.5 \text{ N.m.}$$

(*b*) When the speed of rotation, ω = 250 rpm.

$$e_b = K_b\,.\,\omega = 12 \times 10^{-3} \times 250 = 3V$$

and $$T_m = \frac{K_m(V_{in} - e_b)}{R_a} = \frac{1.25\,(25 - 3)}{2.5} = 11 \text{ N.m.}$$

When the speed of rotation is, ω = 500 rpm

$$e_b = K_b\,.\,\omega = 12 \times 10^{-3} \times 500 = 6V$$

and $$T_m = \frac{K_m(V_{in} - e_b)}{R_a} = \frac{1.25(25 - 6)}{2.5} = 9.5 \text{ N.m.}$$

Ans. (*a*) Torque at the start = 12.5 N.m.

(*b*) Torque at 250 rpm. = 11 N.m.

500 r.p.m. = 9.5 N.m.

Example 2.14. *A stepper motor actuates a arm of a pick and place robot. The step angle of the motor is 10°. For each pulse received from the pulse train source, the motor rotates through a distance of one step angle.*

(*a*) *What is the resolution of stepper motor?*

(*b*) *What is the control resolution and accuracy of rotation?*

(*c*) *How many pulses are required to rotate the motor through four complete revolutions?*

(*d*) *If it is desired to rotate the motor at a speed of 20 rpm what must be the pulse rate generated by the controller?*

Sol. Data:

The step angle, A_s = 10°

(*a*) Resolution of the stepper motor

$$R_s = \frac{A_s}{360°} = \frac{10°}{360°} = 0.027°$$

(*b*) Control resolution $= R_s = 0.027°$

$$\text{Accuracy} \geq \frac{0.027}{2} = 0.0135°$$

(*c*) Pulse counts

$$n = \frac{(N_R)360°}{A_s} = \frac{N_R}{R_s}$$

where N_R = number of revolutions = 4

$\therefore \qquad n = \dfrac{4}{(1/36)} = 144$ pulses.

(*d*) Pulse rate, $\qquad n_r = \dfrac{N_m}{A_s R_s}$

where N_m = number of revolutions per minute = 20 rpm

So, $\qquad n_r = \dfrac{20}{(1/36)} = 720$ pulses/min.

Ans. (*a*) Resolution of stepper motor $= \dfrac{1}{36} = 0.027°$.

(*b*) Control resolution = 0.027°, Accuracy = 0.0135°.

(*c*) Number of pulses for 4 rotations = 144 pulses.

(*d*) Pulse rate for 20 rpm = 720 pulses/min.

Example 2.15. *A stepper motor is used to drive a prismatic joint of a cartesian robot. The motor shaft is connected to a screw shaft with a pitch of 3 mm. The control resolution of 0.6 mm is desired from the controller. Determine*

(*a*) *The number of step angles on the motor to achieve this control resolution.*

(*b*) *The pulse rate required to drive the joint with a linear speed of 75 mm/sec.*

Sol. Data :

The screw pitch, $\qquad p_s = 3$ mm

Control resolution, $\qquad R_c = 0.6$ mm

(*a*) The number of step angles,

The number of step angle of stepper motor,

$$A_s = \frac{360° \cdot R_c}{p_s} = \frac{360° \times 0.6}{3} = 72°$$

The number of step angles $= \dfrac{360}{A_s} = \dfrac{360}{72} = 5$ steps.

Resolution of motor $= R_s = \dfrac{A_s}{360} = \dfrac{72°}{360} = 0.2°$

(*b*) The pulse rate, Velocity = 75 mm/sec.

3 mm of screw movement corresponds to 1 rotation of the shaft. Hence for 75 mm movement

$$N_m = \frac{75}{3} = 25 \text{ rps.}$$

But the pulse rate,

$$n_r = \frac{N_m}{R_s} = \frac{25}{0.2} = 125 \text{ pulse/sec.}$$

Ans. (*a*) Resolution of motor = 0.2°

The number of step angles = 5

(*b*) The pulse rate = 125 pulses/sec.

Example 2.16. *The mechanical gripper uses friction to grasp a part weighing 25 N. The co-efficient of friction between the part and the gripper pad shown in Fig. 2.20 is 0.3. The gripper is accelerating down with a acceleration = 9.81 m/s². The diameter of the piston of pneumatic cylinder is 65 mm. Assume a factor of safety = 1.5 and assume the lengths $L_1 = 60$ mm, $L_2 = 40$ mm, $L_3 = 15$ mm, $L_4 = 45$ mm.*

Calculating the following :

(*a*) *The gripping force to retain the part.*

(*b*) *Actuation force required to achieve this gripping force.*

(*c*) *The pressure of air needed to operate the piston.*

(*d*) *The power required if the discharge is 0.015 m³/sec.*

Sol.

(*a*) The gripping force to retain the part

Using the expression (2.26)

$$P_g = \frac{n \cdot W}{\mu \cdot N}(1 + k_f)$$

where $n = 1.5$, $W = 25$ N

$\mu = 0.3$, $N = 2$

$$k_f = \frac{a}{g} = \frac{9.81}{9.81} = 1.$$

Hence $\qquad P_g = \dfrac{1.5(25)}{0.3(2)}(1 + 1) = 125$ N.

(*b*) Actuation force needed with the use of equation (2.29)

$$P_a = \frac{2nW}{\mu N}\left(\frac{L_1 \cdot L_3}{L_2 \cdot L_4}\right)(1 + k_f)$$

$$= \frac{2(1.5)(25)}{0.3(2)}\left(\frac{60 \times 15}{40 \times 45}\right)(1 + 1) = 125 \text{ N}$$

(*c*) The pressure of the air needed to operate piston using the expression (2.13)

$$p = \frac{4}{\pi}\frac{p}{d_p^{\,2}}$$

where d_p = 65 mm (0.065 m)

$$p = \frac{4}{\pi} \cdot \frac{125}{(65)^2} = 0.0376 \text{ N/mm}^2 \ (3.76 \times 10^5 \text{ N/m}^2)$$

(*d*) The power required

$$P_R = \frac{p \cdot Q}{1000}$$

where volume flow rate, Q = 0.015 m³/sec

$$P_R = \frac{3.76 \times 10^6 \times 0.015}{1000} = 5.65 \text{ kW.}$$

Ans. (*a*) Gripping force = 125 N

(*b*) Actuation force = 125 N

(*c*) Pressure of air = 3.76×10^5 N/m²

(*d*) Power required = 5.65 kW.

EXERCISE

2.1. Enumerate the complete Robot classification. *(VTU–Jan./Feb. 2003)*

2.2. Describe the complete classification of Robotic systems. *(VTU–Jan./Feb. 2004)*

2.3. Define a robot and with a diagram explain the anatomy of a robot. *(VTU–Jan./Feb. 2004)*

2.4. With neat sketches differentiate and highlight the four common types of robot configurations. *(VTU–Jan./Feb. 2003)*

2.5. Explain how the performance of robotic system is studied? *(VTU–Jan./Feb. 2003)*

2.6. Compare three basic types of drives enlisting their merits and demerits. *(VTU–Jan./Feb. 2004)*

2.7. What are the merits and demerits of electric drive system, of a robot. *(VTU–May/June 2004)*

2.8. Discuss the criteria of selection of drive systems for the robots, highlighting the merits and demerits of the system. *(VTU–Jan./Feb. 2003)*

2.9. Discuss briefly about the grippers and give its classification. *(VTU–May/June 2004)*

2.10. Describe with a neat sketch degrees of freedom associated with a robot wrist. *(VTU– May/June 2004)*

2.11. Give a general representation of the robot link, discussing its design considerations.

2.12. With neat sketches, explain different configurations of robot joints.

2.13. Enlist the robot specifications and explain each of them briefly.

2.14. Define repeatability, resolution and accuracy.

2.15. Enumerate the factors that contribute to the limitation of the spatial resolution.

2.16. Derive and explain the total resolution with reference to a cylindrical co-ordinate robot.

2.17. Give a brief classification of actuators used in robots.

2.18. Enlist the main elements of a hydraulic system used in robot and explain their functions briefly.

2.19. With a neat sketch explain the following hydraulic actuator.

(*i*) Rotary actuator. (*ii*) Linear actuator.

2.20. Explain the features and applications of hydraulic actuators in robotics.

2.21. Compare the features of most commonly used electric actuators in robotics.

2.22. Explain the performance and selection criteria of electric motors in robotics.

2.23. Explain with a schematic diagram explain the operating principle of a stepper motor used in robotics.

2.24. With neat sketches, give the classification of the wrist based on the type of motions.

2.25. Explain the features and applications of any two types of wrist.

2.26. Give a brief classification of gripper finger types.

2.27. With neat sketches explain any two types of gripper mechanisms.

2.28. Derive with usual notations the expression for force exerted by the mechanical grippers in robotics.

2.29. Discuss the gripper design considerations in robotics.

2.30. Explain the selection criteria of end-effectors in robotics.

2.31. An industrial Robot with a prismatic joint has a telescoping range of 0.6 m. The robots control memory has the following bit storage capacity.

(*a*) 10 bit storage capacity. (*b*) 12 bit storage capacity.

Determine the control resolution for the two cases separately.

2.32. A cartesian robot with three linear motions has the traversing range of 50 cm, 70 cm and 90 cm with a control memory of 8 bit, 10 bit and 12 bit storage capacity respectively. Determine the total control resolution of the robots work volume.

2.33. A servomotor used to actuate one of the joints of the cylindrical robot has a torque constant of 30 N-m/A and a voltage constant of 15×10^{-3} V/rpm. The armature resistance is 30 Ω. When the joint is stationary the voltage applied is 60 V. Determine

(*a*) The torque exerted by the motor on the joint when the robot is stationary.

(*b*) Determine the back e.m.f. and the torque when the motor is running at 500 rpm and 1000 rpm.

2.34. Explain the construction and working of a pneumatic gripper.

2.35. Explain the construction and working of a multipole stepper motor.

2.36. Sketch and explain single phase stepper motor.

2.37. With a block diagram explain the functioning of a servo-motor.

2.38. What is workspace ? Give the functional diagram with the workspace for the following robots

(*a*) 3R-robot (*b*) 2RP robot

(*c*) R2P robot (*d*) 3P-robot

2.39. Give the functional diagrams for the following robots

(*a*) POMA robot (*b*) Kawasaki Unimate robot

(*c*) I3 robot (*d*) IBM robot

(*e*) SCARA robot.

2.40. Explain the application of Hydraulic system to a robot actuation.

3 Robot Motion Analysis

3.1 INTRODUCTION

The definition of the industrial robot describing the anatomy starts with the explanation of the mechanical manipulator, the motion of the manipulator and the understanding of the robot arm kinematics.

• Mechanical Manipulator

modeled as an open chain of articulation, has many links that are connected by prismatic or revolute joints driven by actuators. The open chain has to have two free ends with the starting end being connected to the robot base, where as the other end accomodates the tool or the end effector, to grab and position the object, or to carry out the operation.

• Manipulation

is the study of the relative motion giving the relationship between the objects themselves or the objects and the manipulator elements.

• Robot Arm Kinematics

explain the analytical description of the motion geometry of the manipulator with reference to a robot co-ordinate system fixed to a frame, without the consideration of the forces or the moments causing the movements. The motion is described as a function of time. For the direct kinematics the inputs are the joint angle vectors and the link length parameters. The output of the problem is the orientation and the position of the tool or the gripper. The block diagram representation of the direct kinematics is as given under in Fig. 3.1.

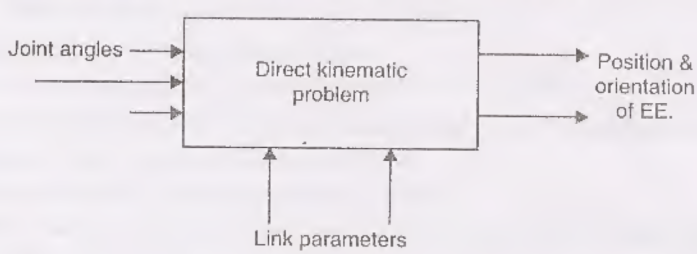


Fig. 3.1 Direct Kinematics.

In certain situations it is possible to know the position and orientation of the objects placed in the work place or envelope and it is desired to know the joint vectors given the link parameters of the robots. Such a problem is known as inverse kinematic problem, represented by the block diagram in Fig. 3.2.

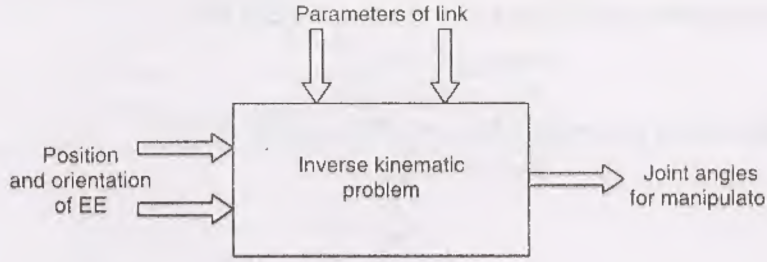


Fig. 3.2

3.2 TRANSFORMATIONS

To describe the position and orientation of the tool with respect to the base frame it is necessary to know and formulate the body attached co-ordinate frame along the joint axis for each links in the manipulator chain of the robot. The relation between the body attached frame with the base frame of reference is described by the transformation matrix. The transformation is represented by the following component transformations.

1. Rotation matrix
2. Translation or position vector
3. Perspective transformation
4. Scaling or stretching.

The transformation matrix is a 4 × 4 matrix which consists of four sub-matrices shown as follows.

$$\left[\begin{array}{c:c} \begin{matrix}\text{Rotation Matrix}\\ (3\times 3)\end{matrix} & \begin{matrix}\text{Position}\\ \text{Vector}\\ (3\times 1)\end{matrix} \\ \hdashline \begin{matrix}\text{Perspective } (1\times 3)\\ \text{Transformation}\end{matrix} & \begin{matrix}\text{Stretching}\\ (1\times 1)\end{matrix} \end{array}\right]$$

3.3 ROTATION MATRIX

A position vector representing a point in the three dimensional space is operated by the rotation matrix to map its co-ordinates in the rotated or body attached reference frame with the base reference frame.

Let body attached reference frame be represented by OABC as shown in the Fig. 3.3 and the base reference frame be OXYZ.

Let the position vector with respect to OABC be p_{ABC} and the same point P has vector representation in the OXYZ frame be p_{XYZ}.

The vector

$$p_{ABC} = p_a i_a + p_b i_b + p_c i_c$$

and the vector

$$p_{XYZ} = p_x i_x + p_y i_y + p_z i_z.$$

The co-ordinates of the point P in the two frames are

$$p_{ABC} = (p_a, p_b, p_c)^T$$
$$p_{XYZ} = (p_x, p_y, p_z)^T$$

The two vectors are related by a rotation matrix 'R' as to

$$p_{XYZ} = R\, p_{ABC} \qquad \text{...(3.1)}$$

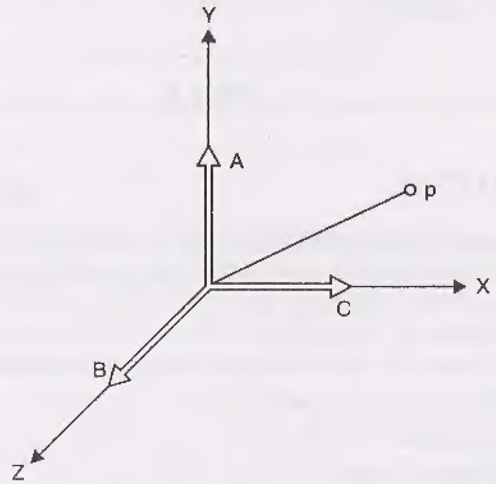


Fig. 3.3

It can also be stated that p_x, p_y, p_z are the components of p along OX, OY and OZ axes, respectively. Given the rotation angle (or rotation matrix) and the vector p_{ABC}, the vector p_{XYZ} can be obtained by using the equation 3.1. But on the other hand p_{XYZ} is known, the position vector p_{ABC} can be obtained by the following expression

$$p_{ABC} = R^{-1}\, p_{XYZ} \qquad \text{...(3.2)}$$

• Rotation about x-axis

To develop a relation for the rotation matrix R(x, α), the body attached co-ordinate system OABC is rotated about OX of the base co-ordinate system by an angle α, which is represented by the diagram Fig. 3.4.

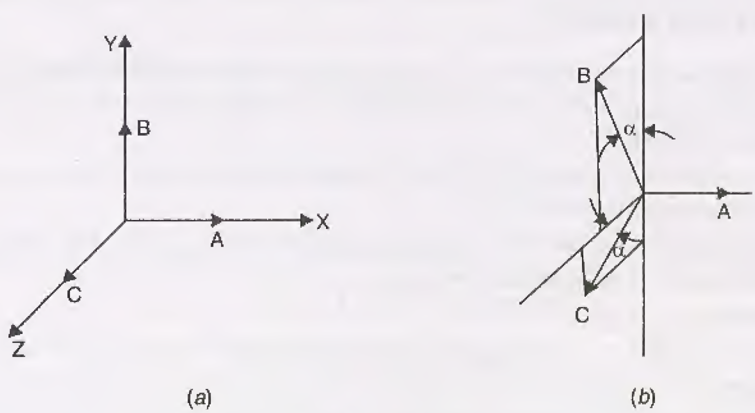


Fig. 3.4

It is evident from the Fig. 3.4(b) that

$$\begin{aligned} p_x &= p_A + 0(p_B) + 0(p_C) \\ p_y &= 0(p_A) + p_B \cdot \cos\alpha + p_C \cdot(-\sin\alpha) \\ p_z &= 0(p_A) + p_B \cdot \sin\alpha + p_C \cdot \cos\alpha. \end{aligned} \quad ...(3.3)$$

The equations (3.3) can be expressed in the matrix form as

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} p_A \\ p_B \\ p_C \end{bmatrix} \quad ...(3.4)$$

or

$$p_{xyz} = R(x, \alpha) \cdot p_{ABC}. \quad ...(3.5)$$

- **Rotation about y-axis**

The Fig. 3.5 shows the vector p_{ABC} being rotated by an angle β about the OY axis of the OXYZ base co-ordinate system. The expression for the rotation transformation matrix, R(y, β) can be

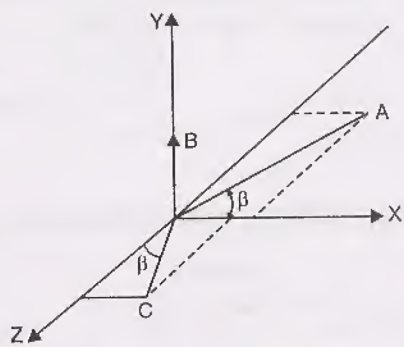


Fig. 3.5

Obtained from the following.

$$\begin{aligned} p_x &= p_A \cdot \cos\beta + p_B(0) + p_C \cdot \sin\beta \\ p_y &= p_A \cdot (0) + p_B(1) + p_C \cdot (0) \\ p_z &= p_A \cdot (-\sin\beta) + p_B(0) + p_C \cdot (\cos\beta) \end{aligned} \quad ...(3.6)$$

The rotation matrix is

$$R(y, \beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad ...(3.7)$$

The expression for p_{xyz}

$$p_{xyz} = R(y, \beta) . p_{ABC}. \quad ...(3.8)$$

- **Rotation about z-axis**

The OABC frame is rotated by an angle θ about the oz-axis of the base frame OXYZ. The position vector p_{xyz} has its components derived out of the vector p_{ABC} as shown in the Fig. 3.6.

$$\begin{aligned} p_x &= p_A \cdot \cos\theta + p_B(-\sin\theta) + p_C \cdot (0) \\ p_y &= p_A \cdot \sin\theta + p_B \cdot \cos\theta + p_C \cdot (0) \\ p_z &= p_A \cdot (0) + p_B(0) + p_C \cdot (1) \end{aligned} \quad ...(3.9)$$

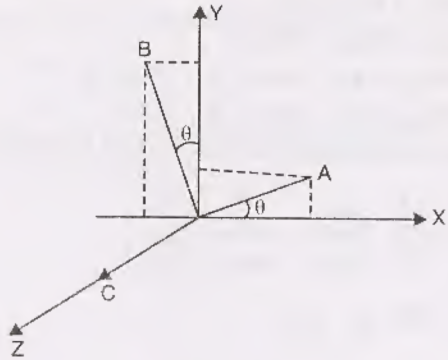


Fig. 3.6

The equation (3.9) can also be expressed as

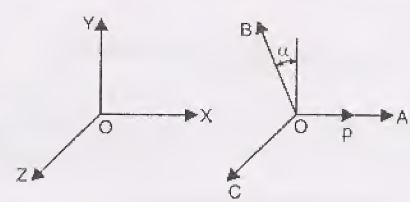
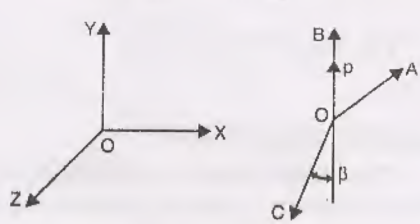
$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_A \\ p_B \\ p_C \end{bmatrix} \qquad ...(3.10)$$

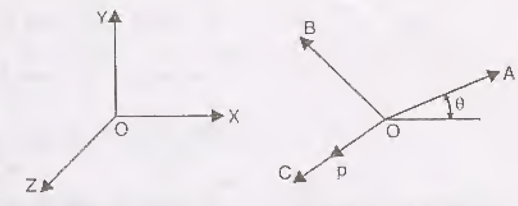
or $$p_{xyz} = R(z, \theta)\, p_{ABC} \qquad ...(3.11a)$$

where $R(z, \theta)$ is the rotation matrix.

3.4 GEOMETRIC INTERPRETATION OF ROTATION MATRIX

Table 3.1

Rotation	*Rotation Matrix*	p_{abc}	*Geometric Representation*
$R(x, \alpha)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	
	The first column/row is same as the unit vector $p_{abc} = (1, 0, 0)^T$ in OABC		
$R(y, \beta)$	$\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\alpha \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	
	The second row/column of R matrix is same as $p_{abc} = (0, 1, 0)^T$ in OABC		

$R(z, \theta)$	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	
The third column/row of R is same as the unit vector $p_{abc} = (0, 0, 1)^T$ in OABC.			

3.5 INVERSE TRANSFORMATIONS

Rotation Matrix

If two co-ordinate frames are orthonormal and are given by X and A, and R to be the transformation that maps X co-ordinate into A co-ordinate,

Then $X = R.A,$

and A maps with X by the relation

$$A = R^{-1}X = R^T X. \quad ...(3.11b)$$

Table 3.2

Rotation	*R matrix*	$R^{-1} = R^T$
1. $R(x, \alpha)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$
2. $R(y, \beta)$	$\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$	$\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$
3. $R(z, \theta)$	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3.6 COMPOSITE ROTATION MATRIX

It is possible to multiply basic rotation matrices in an order of definite rotation angles about the base co-ordinate system's OXYZ, principal axes OX, OY, and OZ. The order in which the multiplication is carried out is important because the multiplication of matrices is not commutative. A sequence of rotation to get a composite rotation matrix is given by

1. A rotation of angle α about OX axis
2. A rotation of angle θ about OZ axis
3. A rotation of angle β about OY axis

$$R_{comp} = R(Y, \beta), R(Z, \theta), R(X, \alpha).$$

$$= \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} c\beta.c\theta & s\beta.s\alpha - c\beta.s\theta.c\alpha & c\beta.s\theta.s\alpha + s\beta.c\alpha \\ s\theta & c\theta.c\alpha & -c\theta.s\alpha \\ -s\beta.c\theta & s\beta.s\theta.c\alpha + c\beta.s\alpha & c\beta.c\alpha - s\beta.s\theta.s\alpha \end{bmatrix} \quad ...(3.12)$$

The simplified notations are

$$c\alpha \equiv \cos\alpha \qquad s\alpha \equiv \sin\alpha$$
$$c\beta \equiv \cos\beta \qquad s\beta \equiv \sin\beta$$
$$c\theta \equiv \cos\theta \qquad s\theta \equiv \sin\theta.$$

The following rules apply in the formation of composite rotation matrix.

- In the start both, reference (base) co-ordinate and the mobile co-ordinate system (OABC) are co-incident, hence the rotation matrix is a (3 × 3) identity matrix, I_3.
- If the mobile co-ordinate system OABC is rotating about one of the axes of fixed co-ordinate system (OXYZ), then '*pre-multiply*' the previous resultant matrix with the current basic rotation matrix.
- If the mobile co-ordinate system (OABC) is rotating about one of its own axes, then '*post-multiply*' the previous resultant matrix with the current basic rotation matrix.

Representation : Sequence of rotation

(*i*) Rotation of OABC by angle ϕ about OY axis.
(*ii*) Rotation of OABC by angle θ about OC axis
(*iii*) Rotation of OABC by angle α about OA axis

$$R = R(y, \phi).\ I_3.\ R(c, \theta).\ R(A, \alpha).$$

$$= \begin{bmatrix} c\phi & 0 & s\phi \\ 0 & 1 & 0 \\ -s\phi & 0 & c\phi \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

$$= \begin{bmatrix} c\phi c\theta & s\phi s\alpha - c\phi s\theta c\alpha & c\phi s\theta s\alpha + s\phi c\alpha \\ s\theta & c\theta c\alpha & -c\theta s\alpha \\ -s\phi c\theta & s\phi s\theta c\alpha + c\phi s\alpha & c\phi c\alpha - s\phi s\theta s\alpha \end{bmatrix} \quad ...(3.13)$$

PROBLEMS

Example 3.1. *The co-ordinate of a paint $p_{abc} = (5, 4, 3)^T$ in the body co-ordinate frame OABC is rotated 30° about OZ-axis. Determine the co-ordinates of the vector p_{xyz} with respect to base reference co-ordinate frame.*

Sol. Data :

$$p_{abc} = (5, 4, 3)^T$$
$$\theta = 30°$$

Using expressions, (3.11) and (3.10)

$$p_{xyz} = R(z, \theta).\ p_{abc}.$$

$$= \begin{bmatrix} \cos(30) & -\sin(30) & 0 \\ \sin(30) & \cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} = [2.33, 5.964, 3]^T.$$

Ans. The co-ordinates of the point p_{xyz} with respect to the base reference co-ordinate frame is given by

$$p_{xyz} = [2.33, 5.964, 3]^T.$$

Example 3.2. *The co-ordinates of a point q_{abc} is given by $(7, 5, 3)^T$ which is rotated about the OX-axis of the reference frame OXYZ, by angle of 60°. Determine the co-ordinates of the point q_{xyz}.*

Sol. Data :

The point $q_{xyz} = ?$

$q_{abc} = (7, 5, 3)^T$

$\alpha = 60°$

Using equations (3.4) and (3.5)

$$q_{xyz} = R(x, 60°).\ q_{abc}.$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60° & -\sin 60° \\ 0 & \sin 60° & \cos 60° \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -0.866 \\ 0 & 0.866 & 0.5 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix} = [7, -0.098, 5.83]^T.$$

Ans. The co-ordinates of the vector $q_{xyz} = \begin{bmatrix} 7 \\ -0.098 \\ 5.83 \end{bmatrix}$

$$= [7, -0.098, 5.83]^T.$$

Example 3.3. *The co-ordinates of a point p_{abc} in the mobile frame OABC is given by $\left[4, 3, 2\sqrt{3}\right]^T$. If the frame OABC is rotated 60° with respect to OY of the OXYZ frame, find the co-ordinates of p_{xyz} with respect to the base frame.*

Sol. Data :

$$p_{abc} = \left[4, 3, 2\sqrt{3}\right]^T$$

$$\beta = 60°$$

The rotation matrix is given by equations (3.7) and (3.8)

$$R(y, \beta) = R(y, 60°)$$

$$= \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 0.5 \end{bmatrix}$$

But using equation (3.8)

$$p_{xyz} = R(y, 60°)\, p_{abc}.$$

$$= \begin{bmatrix} 0.5 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2\sqrt{3} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ -\sqrt{3} \end{bmatrix}.$$

Ans. The co-ordinate of point p with respect to the base frame of reference = $[5, 3, \sqrt{3}]^T$.

Example 3.4. *For the following rotation matrix determine the axis of rotation and the angle of the rotation about the same.*

$$R = \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & \sqrt{3}/2 \end{bmatrix}$$

Sol. Comparing the rotation matrix R with the equation (3.7)

$$R = R(y, \beta)$$

Hence the axis of rotation is OY axis.

But $$R = (y, \beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & \sqrt{3}/2 \end{bmatrix}$$

i.e. $$\cos\beta = \sqrt{3}/2 \text{ and } \sin\beta = 0.5, \tan\beta = 1/\sqrt{3}$$

Hence $$\beta = 30° \text{ or } (\pi + 30°).$$

Ans. Axis of rotation is OY axis and angle of rotation = 30° or 210°.

Example 3.5. *A mobile body reference frame OABC is rotated 60° about OY-axis of the fixed base reference frame OXYZ. If $p_{xyz} = (2, 4, 6)^T$ and $q_{xyz} = (3, 5, 7)^T$ are the co-ordinates with respect to OXYZ plane, what are the corresponding co-ordinates of p and q with respect to OABC frame ?*

Sol. $$p_{xyz} = (2, 4, 6)^T \text{ and } q_{xyz} = (3, 5, 7)^T$$

60° rotation about OY axis of OXYZ.

Using the equation (3.2).

$$p_{abc} = R(y, 60°)^{-1}, p_{xyz} = R(y, 60°)^T p_{xyz}$$

$$q_{abc} = R(y, 60°)^{-1}, q_{xyz} = (y, 60°)^T q_{xyz}$$

But $$R(y, 60°) = \begin{bmatrix} \cos 60° & 0 & \sin 60° \\ 0 & 1 & 0 \\ -\sin 60° & 0 & \cos 60° \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.866 \\ 0 & 1 & 0 \\ -0.866 & 0 & 0.5 \end{bmatrix}$$

and $$R(y, 60°)^T = \begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix}$$

Hence $$p_{abc} = \begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -4.196 \\ 4 \\ 4.732 \end{bmatrix}$$

and $$q_{abc} = \begin{bmatrix} 0.5 & 0 & -0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} -4.562 \\ 5 \\ 6.098 \end{bmatrix}.$$

Ans. (*i*) $p_{abc} = [-4.196, 4, 4.732]^T$

(*ii*) $q_{abc} = [-4.562, 5, 6.098]^T$.

Example 3.6. *The co-ordinates of point Q with respect to base reference frame is given by* $[4, 2\sqrt{3}, 5]^T$. *Determine the co-ordinates of Q with respect to mobile rotated frame of the robot if the angle of rotation with the OX is 60°.*

Sol. Data :

Using the equation (3.11*b*)

$$Q_{abc} = R^{-1}(x, 60°).\ Q_{xyz}$$

But the two co-ordinate frames are orthogonal

$$R^{-1}(x, 60°) = R^T(x, 60°)$$

But $$R(x, 60°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 0.5 \end{bmatrix}$$

$$R^T(x, 60°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 0.5 \end{bmatrix}$$

$$Q_{abc} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 0.5 \end{bmatrix} \begin{bmatrix} 4 \\ 2\sqrt{3} \\ 5 \end{bmatrix} = [4, 1.1\sqrt{3}, 0.5]^T.$$

Ans. The co-ordinates of Q in the mobile frame is

$$Q_{abc} = [4, 1.1\sqrt{3}, 0.5]^T.$$

Example 3.7. *A single axis robot with a fixed base and a mobile link is as shown in the Fig. 3.7. Suppose the mobile frame has a point* p_m *given by* $(2, 2, 8)^T$. *Find the co-ordinates of the the point* p_F *with respect to base frame when* $\theta_1 = 180°$ *and* $\theta_2 = 0°$.

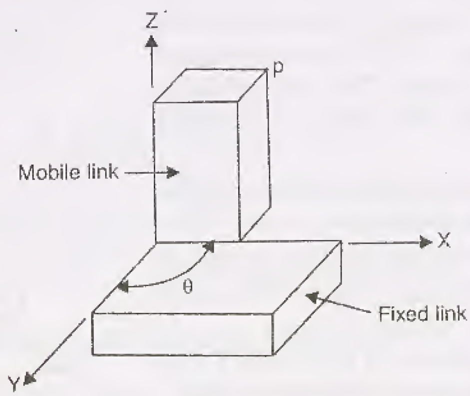


Fig. 3.7

Sol. Data :

The co-ordinates of point p in mobile frame,

$$p_M = [2, 2, 8]^T$$
$$\theta_1 = 180°, \theta_2 = 0°.$$

Using the equation (3.10)

$$R(z, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(*i*) When $\theta_1 = 180°$

$$R(z, \theta_1) = \begin{bmatrix} \cos 180° & -\sin 180° & 0 \\ \sin 180° & \cos 180° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(p_F)_{\theta = 180°} = R(z, 180°)\,[p_M]$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 8 \end{bmatrix}$$

(*ii*) When $\theta_2 = 0°$

$$R(z, \theta_2) = \begin{bmatrix} \cos 0° & -\sin 0° & 0 \\ \sin 0° & \cos 0° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(p_F)_{\theta = 0°} = R(z, \theta_2),\,[p_M]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix}.$$

Ans : The co-ordinates of p_F in the fixed frame.

(*i*) $(p_F)_{\theta = 180°} = [-2, -2, 8]^T$

(*ii*) $(p_F)_{\theta = 0°} = [2, 2, 8]^T$.

Example 3.8. *A robotic end effector is positioned as shown in Fig. 3.8. The yaw of the end effector (EE) is $\pi/2$ about z-axis. The pitch of EE is π, about X-axis and the roll of the EE is $-\pi/2$, about Y-axis.*

(a) Draw the sketch of the EE in sequence after each of yaw, pitch and roll motions.

(b) The composite transformation matrix T which maps the tip co-ordinates into the EE's wrist frame.

(c) Find the co-ordiantes of a point p(0, 1.6, 0) at tool tip with respect to wrist co-ordinate frame.

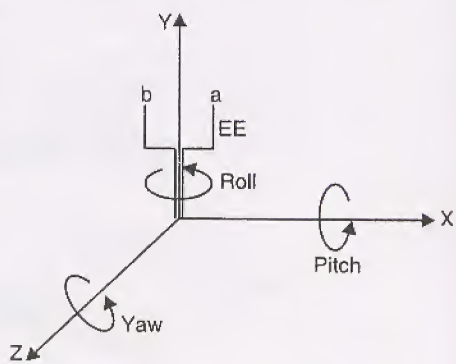
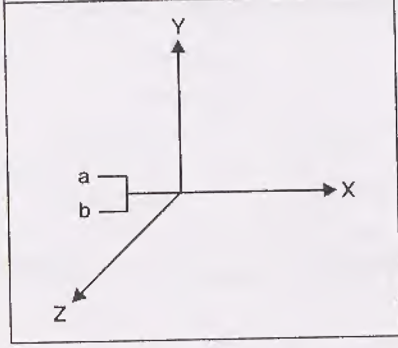
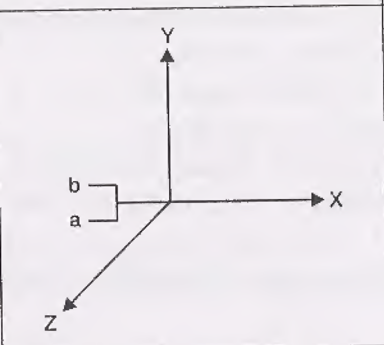
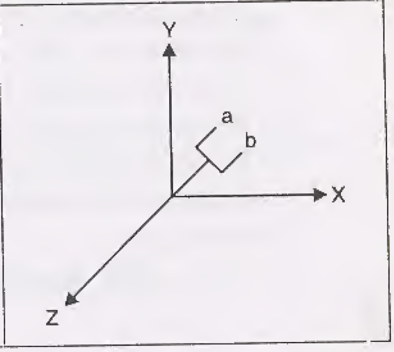


Fig. 3.8

Sol.

(*a*) Tool's positions

Yaw : π/2 about z	*Pitch : π about x*	*Roll : – π/2 about Y*
		

(*b*) The composite transformation matrix

$$T = R(y, -\pi/2), R(x, \pi), R(z, \pi/2)$$

$$= \begin{bmatrix} \cos(-\pi/2) & 0 & \sin(-\pi/2) \\ 0 & 1 & 0 \\ -\sin(\pi/2) & 0 & \cos(-\pi/2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi & -\sin\pi \\ 0 & \sin\pi & \cos\pi \end{bmatrix} \begin{bmatrix} \cos\pi/2 & -\sin\pi/2 & 0 \\ \sin\pi/2 & \cos\pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow \text{Composite transformation matrix.}$$

(*c*) The position vector.

$$p_w = T.p_t$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1.6 \\ 0 \end{bmatrix} = [0, 0, -1.6]^T.$$

Ans. The co-ordinates of the tool tip after transformation with respect to wrist.

$$p_w = [0, 0, -1.6]^T.$$

3.7 ROTATION MATRIX ABOUT AN ARBITRARY AXIS

The mobile co-ordinate frame OABC is rotated through an angle θ, about a vector V with components V_x, V_y, V_z. The rotation matrix for a case of rotation about V is analysed in the following manner, represented by the Fig. 3.9.

Let Base frame $\Rightarrow$ OXYZ

Mobile frame $\Rightarrow$ OABC

Arbitrary axis, $V = V_x i + V_y j + V_z k$

Angle of rotation about $V = \theta$.

The rotation matrix $R(V, \theta)$ has the proceding steps of rotation made easy to understand.

(*i*) Rotation by angle, α about *x*-axis, $R(x, \alpha)$

(*ii*) Rotation by angle, $(-\beta)$ about *y*-axis, $R(y, -\beta)$

(*iii*) Rotation by angle, θ about *z*-axis, $R(z, \theta)$

(*iv*) Rotation (reverse) by angle, β about *y*-axis, $R(y, \beta)$

(*v*) Rotation (reverse) by angle, $(-\alpha)$ about *x*-axis, $R(x, -\alpha)$

Hence we have

$$R(V, \theta) = R(x, -\alpha)\, R(y, \beta)\, R(z, \theta)\, R(y, -\beta)\, R(x, \alpha) \qquad ...(3.14)$$

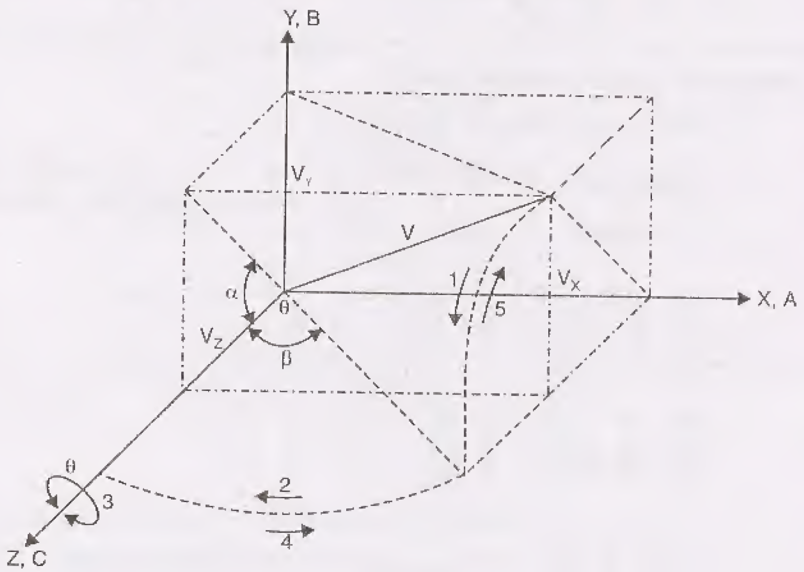


Fig. 3.9 Rotation about Arbitrary Axis.

The resultant rotation matrix about V is $R(V, \theta)$

$$R(V, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & s\alpha \\ 0 & -s\alpha & c\alpha \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\beta & 0 & -s\beta \\ 0 & 1 & 0 \\ s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

From the geometry of the Fig. 3.9 it is arrived at the following expression for

$$\sin\alpha = s\alpha = \frac{V_y}{\sqrt{V_y^2 + V_z^2}}, \qquad c\alpha = \cos\alpha = \frac{V_z}{\sqrt{V_y^2 + V_z^2}},$$

$$\sin\beta = s\beta = V_x \qquad \text{and} \qquad \cos\beta = c\beta = \sqrt{V_y^2 + V_z^2}$$

And the rotation matrix resulting from the above is

$$R(V, \theta) = \begin{bmatrix} V_x^2(1-c\theta)+c\theta, & V_xV_y(1-c\theta)-V_z\,s\theta, & V_xV_z(1-c\theta)+V_y\,s\theta \\ V_xV_y(1-c\theta)-V_z\,s\theta, & V_y^2(1-c\theta)+c\theta, & V_yV_z(1-c\theta)-V_x\,s\theta \\ V_xV_z(1-c\theta)-V_y\,s\theta, & V_yV_z(1-c\theta)+V_x\,s\theta, & V_z^2(1-c\theta)+c\theta \end{bmatrix} \quad ...(3.15)$$

where, $c\theta = \cos\theta$,

$s\theta = \sin\theta$.

Example 3.9. *Compute the rotation matrix to represent a rotation of 90° about an arbitrary vector $V = (2, 2, 2)^T$.*

Sol. Data :

$$R(V, \phi) = R(V, 90°) \text{ and } V = (2, 2, 2)^T$$

The components of the vector V about principal axes of the OXYZ frame, are

$$V_x = \frac{(2)^2}{\sqrt{V_x^2+V_y^2+V_z^2}} = \frac{2}{\sqrt{3}}, \quad V_y = \frac{2}{\sqrt{3}} \text{ and } V_z = \frac{2}{\sqrt{3}}.$$

But $$R(V, 90°) = \begin{bmatrix} n_x & s_x & q_x \\ n_y & s_y & q_y \\ n_z & s_z & q_z \end{bmatrix}$$

Comparing with equation (3.15)

$$n_x = V_x^2\,(1-\cos\phi)+\cos\phi = \frac{4}{3}(1-0)+0 = \frac{4}{3}$$

$$n_y = V_x\,V_y(1-\cos\phi)+V_z\,\sin\phi = \frac{2}{\sqrt{3}}.\frac{2}{\sqrt{3}}(1-0)+\frac{2}{\sqrt{3}}(1) = \left(\frac{4}{3}+\frac{2}{\sqrt{3}}\right)$$

$$n_z = V_x\,V_z(1-\cos\phi)-V_y\,\sin\phi = \frac{2}{\sqrt{3}}.\frac{2}{\sqrt{3}}(1-0)-\frac{2}{\sqrt{3}}(1) = \left(\frac{4}{3}-\frac{2}{\sqrt{3}}\right)$$

$$s_x = V_xV_y(1-\cos\phi)-V_z\,\sin\phi = \frac{4}{3}(1-0)-\frac{2}{\sqrt{3}}(1) = \left(\frac{4}{3}-\frac{2}{\sqrt{3}}\right)$$

$$s_y = V_y^2(1-\cos\phi)+\cos\phi = \frac{4}{3}(1-0)+0 = \frac{4}{3}$$

$$s_z = V_yV_z(1-\cos\phi)+V_x\,\sin\phi = \frac{4}{3}(1-0)+\frac{2}{\sqrt{3}}(1) = \frac{4}{3}+\frac{2}{\sqrt{3}}$$

$$q_x = V_xV_z(1-\cos\phi)+V_y\,\sin\phi = \frac{4}{3}(1-0)+\frac{2}{\sqrt{3}}(1) = \frac{4}{3}+\frac{2}{\sqrt{3}}$$

$$q_y = V_yV_z(1-\cos\phi)-V_x\,\sin\phi = \frac{4}{3}(1-0)-\frac{2}{\sqrt{3}}(1) = \frac{4}{3}-\frac{2}{\sqrt{3}}$$

$$q_z = V_z^2(1-\cos\phi)+\cos\phi = \frac{4}{3}(1-0)+0 = \frac{4}{3}$$

Hence,

$$\textbf{Ans. } R(V, 90°) = \begin{bmatrix} \frac{4}{3} & \frac{4}{3}-\frac{2}{\sqrt{3}} & \frac{4}{3}+\frac{2}{\sqrt{3}} \\ \frac{4}{3}+\frac{2}{\sqrt{3}} & \frac{4}{3} & \frac{4}{3}-\frac{2}{\sqrt{3}} \\ \frac{4}{3}-\frac{2}{\sqrt{3}} & \frac{4}{3}+\frac{2}{\sqrt{3}} & \frac{4}{3} \end{bmatrix}.$$

3.8 EULER ANGLE REPRESENTATION

The rigid body rotations of a body in space is represented by a rotation matrix consisting of nine elements due to three rotations provided by three angles ϕ, θ and ψ, known as Euler angles.

Table 3.3. Euler Angles

Number		*Representation I*		*Representation II*		*RPY System*	
		Angle	Axis	Angle	Axis	Angle	Axis
Rotation	1	ϕ	OZ	ϕ	OZ	ψ (Roll)	OX
Sequence	2	θ	OA	θ	OB	θ (Pitch)	OY
	3	ψ	OC	ψ	OC	ϕ (Yaw)	OZ
Graphical representation		Fig. 3.10		Fig. 3.11		Fig. 3.12	
Rotation matrix		$R(\phi, \theta, \psi) = R(z, \phi)\, R(A,\theta)\, R(C, \psi)$		$R(\phi, \theta, \psi) = R(z, \phi)\, R(B, \theta)\, R(C, \psi)$		$R(\psi, \theta, \phi) = R(x,\psi)\, R(y, \theta)\, R(z, \phi)$	

System I of Euler Angles

$$R(\phi, \theta, \psi)_I = R(z, \phi)\, R(A, \theta)\, R(C, \psi)$$

$$= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $c\phi = \cos\phi$, $c\theta = \cos\theta$, $c\psi = \cos\psi$,

$s\phi = \sin\phi$, $s\theta = \sin\theta$, $s\psi = \sin\psi$,

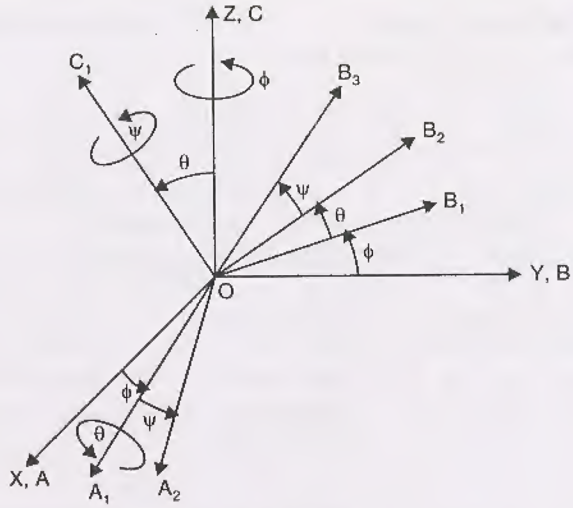


Fig. 3.10. System I Euler Angles.

$$R(\phi, \theta, \psi)_I = \begin{bmatrix} c\phi c\psi - s\phi c\theta s\psi & -c\phi s\psi - s\phi c\theta c\psi & s\phi s\theta \\ s\phi c\psi + c\phi c\theta s\psi & -s\phi s\psi + c\phi c\theta c\psi & -c\phi s\theta \\ s\theta s\psi & s\theta c\psi & c\theta \end{bmatrix} \quad ...(3.16)$$

System II of Euler Angles

$$R(\phi, \theta, \psi)_{II} = R(z, \phi)\, R(B, \theta)\, R(C, \psi)$$

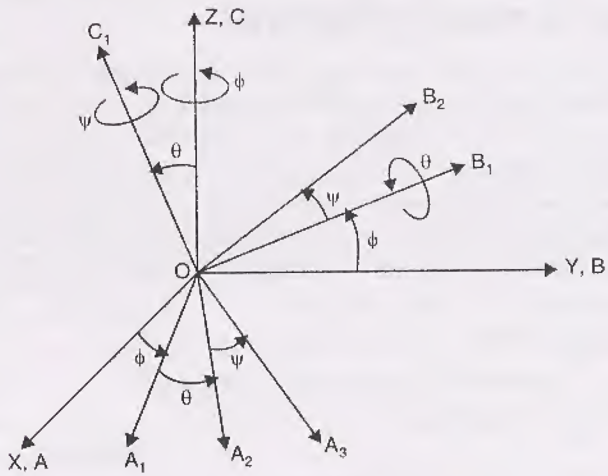


Fig.3.11 System II Euler Angles.

$$= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi s\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi s\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} \quad ...(3.17)$$

Roll-pitch-Y$_{aw}$ System of Euler Angle

$$R(\psi, \theta, \phi) = R(z, \phi)\, R(Y, \theta)\, R(X, \psi)$$

$$= \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}$$

$$= \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \quad ...(3.18)$$

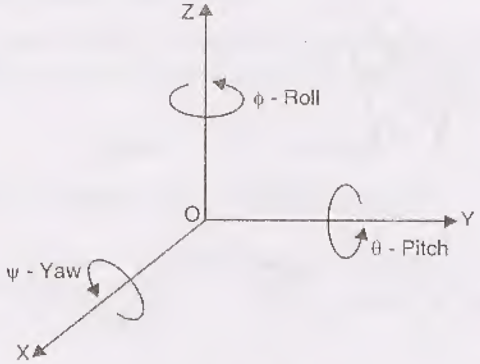


Fig. 3.12 System III (Roll-pitch-Yaw).

3.9 HOMOGENEOUS TRANSFORMATION

A robotic manipulator comprises of joints with both rotational and translatory movements. The previously deescribed rotational matrix does not have provision for linearly moving joint configurations. To accomodate the translation along the position vector $p = (p_x, p_y, p_z)^T$ it is necessary express the 3 × 1 vector in a 4 × 1 homogeneous co-ordinates *i.e.* $(wp_x, wp_y, wp_z, w)^T$. Thus the N component position vector is transformed into (N + 1) component vector by the "homogeneous co-ordinate representation". The physical 'N' dimensional vector is obtained by dividing the homogeneous co-ordinates by the $(N + 1)^{th}$ component w

Physical position vector $= (p_x, p_y, p_z)^T$

Augmented position vector 1 $= (w_1p_x, w_1p_y, w_1p_z, w_1)^T$

vector 2 $= (w_2p_x, w_2p_y, w_2p_z, w_2)^T$

The physical vector is obtained by

$$p_{xyz} = \frac{w_n p_{xyz}}{w_n} \quad ...(3.19)$$

where w_n is the scaling or stretching component. In robotics the scaling factor is always unity (1).

Combining the rotation matrix (3 × 3), the homogeneous translational vector (4 × 1) and a perspective matrix (1 × 3) a homogeneous transformation matrix (4 × 4) is arrived at.

Homogeneous Transformation Matrix

$$\underset{(4 \times 4)}{H =} \left[\begin{array}{c|c} \text{Rotation Matrix} & \text{Position Vector} \\ (3 \times 3) & (3 \times 1) \\ \hline \text{Perspective Transformation} & \text{scale factor} \\ (1 \times 3) & (1 \times 1) \end{array}\right]$$

Table 3.4. Components of Homogeneous Transformation

Type of Transformation	*Portion Spec.*	*Size*	*Application*
Rotational Transformations	Upper Left	(3 × 3)	Revolute joint
Translational vector	Upper Right	(3 × 1)	Prismatic joints
Stretching or scaling transformation	Lower Right	(1 × 1)	Global scaling in computer Graphics
Perspective Transformations	Lower Left	(1 × 3)	Computer vision and camera calibrations.

Transformation Map

The homogeneous transformation matrix maps the mobile co-ordinate frame vector, p_{abc} to the vector p_{xyz} in the base reference frame *oxyz* by,

$$p_{xyz} = \mathrm{H}\, p_{abc} \qquad \text{...(3.20)}$$

where, the general form of H

$$\mathrm{H} = \begin{bmatrix} n_x & s_x & q_x & p_x \\ n_y & s_y & q_y & p_y \\ n_z & s_z & q_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 3.5. Basic Homogeneous Transformations

No.	*Transformations*	*Symbol*	*Matrix Representation*	*Equation No.*
1.	Basic Homogeneous rotation about OX-axis by α angle	H(x, α)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	(3.21)
2.	Basic Homogeneous rotation about OY-axis by angle ϕ	H(y, ϕ)	$\begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	(3.22)
3.	Basic Homogeneous rotation about OZ-axis by angle θ	H(z, θ)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	(3.23)

4.	Basic Homogeneous translation by a vector p_{xyz}	$H_{(trans)}$	$\begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	(3.24)

The first three principal diagonal elements produce local scaling and the last diagonal element is responsible for global stretching or scaling. In robotics the last diagonal element is always one (1).

3.10 GEOMETRIC INTERPRETATION OF HOMOGENEOUS TRANSFORMATION

$$H = \begin{bmatrix} n_x & s_x & q_x & p_x \\ n_y & s_y & q_y & p_y \\ n_z & s_z & q_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The homogeneous transformation matrix H can be geometrically interpreted in line with the rotation matrix, R. The column vectors of rotation sub-matrix (3 × 3) represent the orientation of the principal axes of OABC with respect to the reference frame OXYZ. The fourth-column matrix of H represent the position of the origin of OABC with respect to OXYZ reference frame. In robotics the frame OABC is considered to be mobile frame and OXYZ is considered to be the fixed base frame. The homogeneous transformation matrix maps the points in the OABC frame with the reference frame OXYZ.

i.e., $$p_{xyz} = H.\, p_{abc}\,.$$

Given p_{xyz} the p_{abc} can be obtained by

$$p_{abc} = H^{-1}\, p_{xyz}\,.$$

Here the column matrix of H^{-1} represent the orientation of the reference frame with respect to rotated mobile frame.

3.11 INVERSE HOMOGENEOUS TRANSFORMATIONS

Let 'H' be the homogeneous transformation matrix with rotation R and translation p between two orthonormal co-ordinate frames without perspective transformation and scaling equal to unity. 'H' is given by

$$H = \left[\begin{array}{ccc|c} & R(3 \times 3) & & \begin{matrix} p \\ (3 \times 1) \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

The inverse of H is given by

$$H^{-1} = \left[\begin{array}{ccc|c} & R^T & & -R^T p \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Rotation of angle θ about Z-axis.

$$H(z, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & p_x \\ \sin\theta & \cos\theta & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H^{-1}(z, \theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -(p_x\cos\theta + p_y\sin\theta) \\ -\sin\theta & \cos\theta & 0 & -(-p_x\sin\theta + p_y\cos\theta) \\ 0 & 0 & 1 & -p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.12 COMPOSITE HOMOGENEOUS TRANSFORMATIONS

The position of the end-effector of a robot in a required point in work-space, is achieved through set of rotation and translation by the links about the respective joints. Hence arriving at a composite homogeneous transformation matrix is necessory. Since the matrix multiplication is non-commutative care to be taken in the composite transformations. The following rules find use in the process.

- To begin with both the fixed and mobile co-ordinate systems are coincident. Hence the homogeneous transformation matrix is a identity matrix I_4.
- If the mobile frame, OABC is rotating/translating about one of the axes (OXYZ) of fixed frame '*pre-multiply*' previous resultant matrix with the current homogeneous rotation/translation matrix.
- If the mobile co-ordinate system, OABC, is rotating/translating about one of its own axes, then '*post-multiply*' the resultant (previous) matrix by the present homogeneous transformation matrix.

$$H = H(y, \beta).\ I_4.\ H(A, \alpha)\ H(C, d) \quad \ldots(3.25)$$

$H(Y, \beta)$ = basic rotation about OY by angle β (pre-multiplied)

$H(A, \alpha)$ = basic rotation about OA by angle α (pre-multiplied)

$H(C, d)$ = basic translation along OC by a distance 'd',

I_4 = Idendity matrix, (Post-multiplied).

Example 3.10. *A point $p_{abc} = (2, 3, 4)^T$ has to be translated through distance of + 4 units along OX-axis and – 2 units along OZ-axis. Determine the co-ordinates of the new point p_{xyz} by homogeneous transformation.*

Sol. Data :

$$p_{abc} = (2, 3, 4)^T$$

Homogeneous $\quad p_{abc} = (2, 3, 4, 1)^T$

Position vector $\quad = (4, 0, -2, 1)^T.$

$$H_{tran} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the expression (3.20)

$$p_{xyz} = H_{tran}.p_{abc}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} (1)(2) + (4)(1) \\ (1)(3) + 0 \\ (1)(4) + (-2)(1) \\ (1)(1) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \end{bmatrix}.$$

Ans. $p_{xyz} = [6, 3, 2]^T$.

Example 3.11. *Determine the homogeneous transformation matrix to represent a rotation of 30° about OX-axis and a translation of 8 units along the OB-axis of the mobile frame.*

Sol. $H(x, \alpha) = H(x, 30°)$

$H(B, \beta) = H(B, 8)$

Using the equation (3.25)

$$H = H(x, 30°).I_4.H(B, 8).$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° & 0 \\ 0 & \sin 30° & \cos 30° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -0.5 & 0 \\ 0 & 0.5 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. $$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -0.5 & 4\sqrt{3} \\ 0 & 0.5 & \sqrt{3}/2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Example 3.12. *A triangular prism with co-ordinates of its vertices indicated relative to the fixed reference frame OXYZ, is shown in Fig. 3.13. The prism is moved to the new position with a rotation of + 90° about x-axis, a rotation of – 90° about z-axis and a translation of 5 units in the y-direction.*

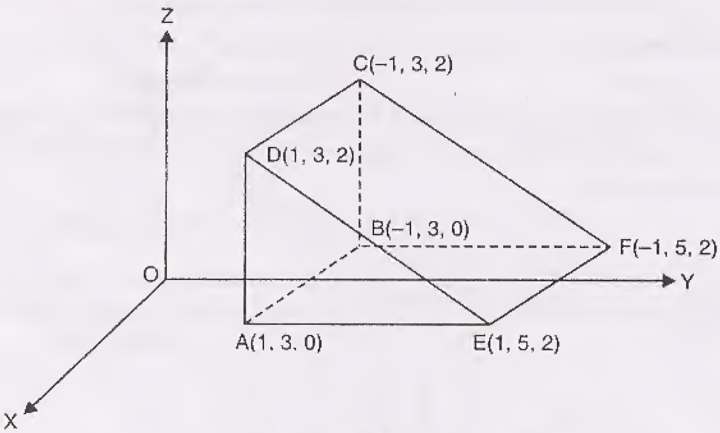


Fig. 3.13. Prism.

Determine

(*i*) The homogeneous transformation describing the change in position of the prism.

(*ii*) The new co-ordinates of the vertices of the prism.

Sol. The sequence of operations leading to homogeneous transformation.

- A rotation of + 90° about x-axis
- A rotation of – 90° about z-axis
- A translation of 5 units in the y-direction. The transformation matrix is

$$H = H_t(0, 5, 0)\, H(z, -90°)\, H(x, +90°).\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 05 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90°) & -\sin(-90°) & 0 & 0 \\ \sin(-90°) & \cos(-90°) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90° & -\sin 90° & 0 \\ 0 & \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 05 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 05 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 05 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- New co-ordinates for A.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix}$$

A = [H] [q]

$$\begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

$[A] = [0, 4, 3, 1]^T$

- New co-ordinates for B.

[B] = [H] [b]

$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

$[B] = [0, 6, 3, 1]^T$

- New co-ordinates for C

[C] = [H] [c]

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = [-2, 6, 3, 1]^T$$

- New co-ordinates for D

[D] = [H] [d]

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = [-2, 4, 3, 1]^T$$

- New co-ordinates for E.

$$[E] = [H]\,[e]$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 2 \\ 1 \end{bmatrix} = [-2, 4, 5, 1]^T$$

- New co-ordinates for F

$$[F] = [H]\,[f]$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \\ 2 \\ 1 \end{bmatrix} = [-2, 6, 5, 1]^T.$$

Ans. (*i*) Homogeneous transformation matrix $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(*ii*) Changed co-ordinates

for vertex A = [0, 4, 3]

for vertex B = [0, 6, 3]

for vertex C = [– 2, 6, 3]

for vertex D = [– 2, 4, 3]

for vertex E = [– 2, 4, 5]

for vertex F = [– 2, 6, 5].

Example 3.13. *Determine the homogeneous transformation matrix to represent the following sequence of operations.*

(*i*) *Rotation of 60° OX-axis.*

(*ii*) *Translation of 4 units along OX-axis*

(*iii*) *Translation of – 6 units along OC-axis*

(*iv*) *Rotation of 30° about OB-axis.*

Sol.

$$H(x, 60°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -\sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H(x, 4) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H(C, -6) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H(B, 30°) = \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By using the expression (3.25)

$$H = H(x, 4)\, H(x, 60°)\, I_4\, H(C, -6)\, H(B, 30°)$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & -\sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\textbf{Ans.} \qquad H = \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 & 4.0 \\ \sqrt{3} & 0.5 & -0.75 & 3\sqrt{3} \\ -0.25 & \sqrt{3}/2 & \sqrt{3}/4 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3.14. *For the object shown in Fig. 3.14 find the 4 × 4 homogeneous transformation matrices ^0A_i for i = 1, 2 and thus find the $^1A_2$ i.e., the transformation of frame at point 2 with respect to the frame at point 1.*

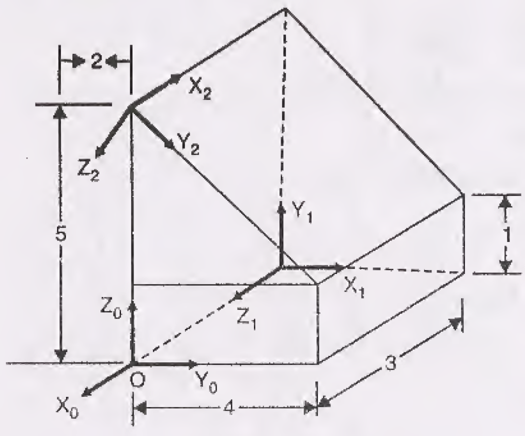


Fig. 3.14

Sol. (*i*) Determination of ^0A_i matrices :

For $i = 1$ $\quad ^0A_i = {}^0A_1$

The following sequences of operations to be performed to get OA_1

- Rotation about z_0 by (– 90°)
- Rotation of (– 90°) about y_0
- Translation (– 3, 0, 0).
- $^0A_1 = H_{trans}(-3, 0, 0)\, R(z_0, -90°)\, R(Y_0, -90°)$

$$= \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $i = 2$ $^0A_1 = {}^0A_2$

Transformation $^0A_2$ requires following operational sequence.

- Rotation by 180° about Y_0.
- Rotation by 45° about X_0.
- Translation of (0, 0, 5)

$^0A_2 = H_{trans}(0, 0, 5)\ R(Y_0, 180°)\ R(X_0, 45°).$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0A_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*ii*) To determine $\quad ^1A_2 = [^0A_1]^{-1}\ [^0A_2]$

$$^1A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans.
$$^1A_2 = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 5 \\ -1 & 0 & 0 & 3 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Example 3.15. *Write down the homogeneous transformation matrices for the co-ordinate frames situated at the points A, B and C, with respect to base co-ordinate frame 0. What is the position and orientation of B with respect to frame 'C' ? Refer Fig. 3.15 for the considered co-ordinate frames.*

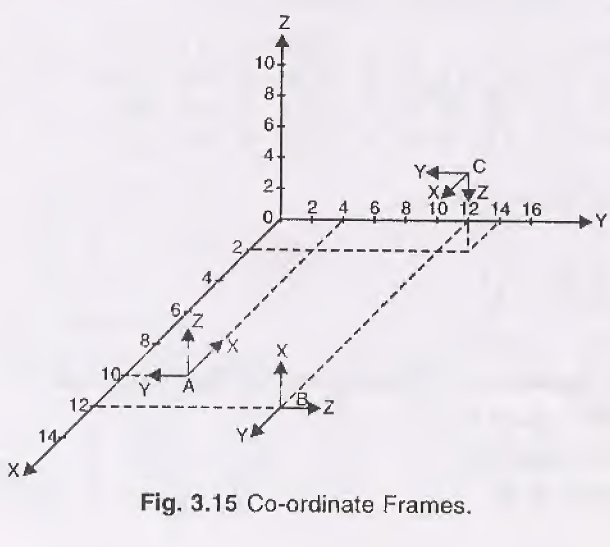


Fig. 3.15 Co-ordinate Frames.

Sol. (*i*) Homogeneous transformation matrices.

(*a*) $$^0H_A = H_{trans}(10, 4, 0)\, H(Z, 180°) I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 180 & -\sin 180 & 0 & 0 \\ \sin 180 & \cos 180 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*b*) $$^0H_B = H_{trans}(12, 12, 0)\, H(Y, +90°)\, H(Z, +90)\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 12 \\ 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*c*) $$^0H_C = H_{trans}(2, 14, 5)\, H(x, 180°)\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 14 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*ii*) To determine the position of B w.r.t. C

$$^CH_B = [^0H_C]^{-1}\, [^0H_B]$$

$$[^0H_C]^{-1} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & +14 \\ 0 & 0 & -1 & +5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^CH_B = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & +14 \\ 0 & 0 & -1 & +5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 12 \\ 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 10 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The position vector = $[10, 2, 5]^T$

$$\text{The orientation vectors} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Example 3.16. *Write down the homogeneous transformation matrices for the co-ordinate frames situated at the points A, B and C with respect to $OX_0Y_0Z_0$ frame. Write down by inspection and matrix operation the position and orientation of frame B with respect to frame C. (Refer Fig. 3.16).*

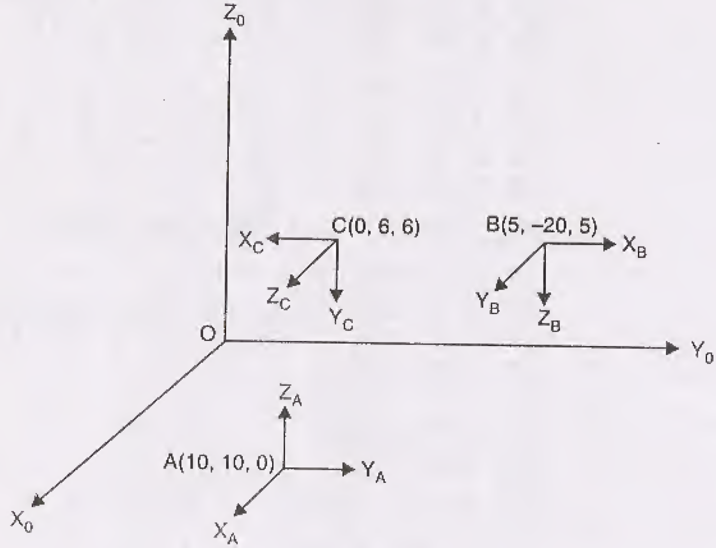


Fig. 3.16

Sol.

(*a*) Homogeneous transformation matrix for point A. The following operation to be performed.

Translation through (10, 10, 0)

$$^0H_A = H_{trans}(10, 10, 0)\, I_4.$$

$$= \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*b*) Homogeneous Transformations for point B

To reach point B following sequence of operations to be performed.

- Rotation about Y_0 by an angle – 180°
- Rotation about Z_0 by an angle 90°
- Translation to a position (5, 20, 5)

Hence $\quad ^0H_B = H_{trans}(5, 20, 5)\, R(Y_0, -180°)\, R(Z_0, +90°)\, I_4$

$$= \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & +1 & 0 & 5 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*c*) Homogeneous Transformation matrix for C. To attain point C, the following operations are to be performed.

- Rotation of – 90° about Y_0
- Rotation of 90° about X_0
- Translation by (0, 6, 6) from origin.

Hence $^0H_C = H_{trans}(0, 6, 6)\, R(X_0, +90)\, R(Y_0, -90°)\, I_4$

$$^0H_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 6 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*d*) Position and orientation of B with respect to C

$$^CH_B = [^0H_C]^{-1}\, {^0H_B}.$$

$$= \begin{bmatrix} 0 & -1 & 0 & +6 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[^CH_B] = \begin{bmatrix} -1 & 0 & 0 & -14 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3.17. *A robotic work cell has a camera with in the setup. The origin of the six joint robot fixed to a base can be seen by the camera. A cube placed on the work cell-table is also seen by the camera. The homogeneous transformation matrix T_1 maps the camera with the cube centre. The origin of the base co-ordinate system as seen from the camera is represented by the homogeneous transformation system T_2.*

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textit{and} \quad H_2 = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*a*) *What is the position and orientation of the cube with respect to the base co-ordinate system ?*

(*b*) *After the system has been setup, some one rotates the camera 90° about the z-axis of the camera. What is the position and orientation of the camera with respect to robot's base co-ordinate system ?*

(*c*) *The same person rotated by 90° the object about the x-axis of the object and translated 5 units of distance along the rotated y-axis. What is the position and orientation of the object with respect to the robot's base co-ordinate system ?*

Sol.

(*a*)
$$^{\text{camera}}H_{\text{cube}} = H_1 = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and
$$^{\text{camera}}H_{\text{base}} = H_2 = \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -1 & 0 & 15 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is required to find $^{\text{base}}H_{\text{cube}}$. By 'chain product' rule

$$^{\text{base}}H_{\text{cube}} = {}^{\text{base}}H_{\text{camera}} \cdot {}^{\text{camera}}H_{\text{cube}} = (H_2)^{-1} H_1$$

$$= \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & -1 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & -23 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Ans. Position of cube is given by [10, – 23, – 13]

The orientation $[n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(*b*) Camera is rotated by 90° about the z-axis of the camera.

$$^{\text{base}}H_{\text{camera}} = (H_2)^{-1}.H(z, 90°)_{\text{camera}}.$$

$$H(z, 90°)_{\text{camera}} = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{\text{base}}H_{\text{camera}} = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & -1 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 8 \\ -1 & 0 & 0 & -15 \\ 0 & 0 & -1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Ans. Position of the camera after the change is given by $[8, -15, -6]^T$

The orientation of camera with respect to base

$$[n, s, a] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(*c*) Now the object is rotated by 90° about x-axis of the object and translated by 5 unit distances along the rotated y-axis of the object.

$$^{b}H_{c} = {}^{\text{base}}H_{\text{cube}}.\ H(x, 90°).H(y, 5)$$

i.e.,
$$^{b}H_{c} = \begin{bmatrix} 0 & 1 & 0 & 10 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans. The position of the object with respect to the base = $[10, 7, 4]^T$

The orientation, $[n, s, a] = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$

Example 3.18. *Write down the homogeneous transformation matrices for the co-ordinate frames attached to the corners A, B, C and D with respect to the base co-ordinate frame '0'. Also write down the transformation matrix for A with respect to 'C' frame and verify the same by finding the inverse.*

The object frame is as shown in Fig. 3.17. (VTU-Jan.Feb. 2004)

Sol. (*i*) Homogeneous transformation matrix for A.

The transformation involves following sequence of operation

- Rotation by 90° about X_0
- Rotation by 180° about Y_0
- Translation by (0, 45, 20) from origin

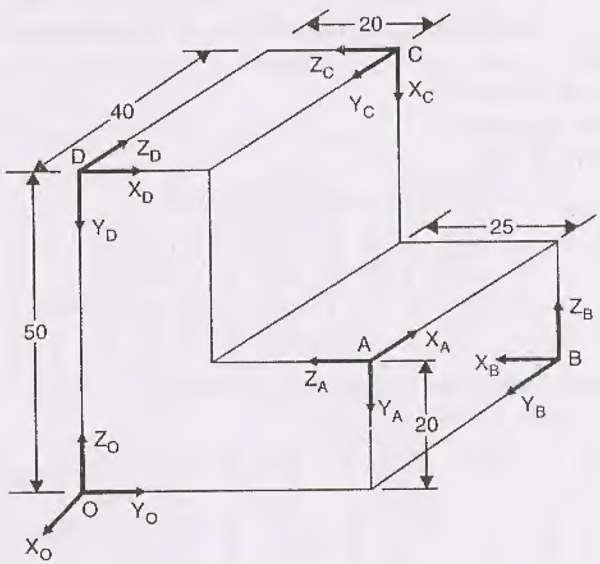


Fig. 3.17 Co-ordinate Frame.

$$^0H_A = H_{trans}(0, 45, 20)\, R(Y_0, 180°)\, R(X_0, 90°)\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 45 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 45 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 45 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*ii*) Homogeneous transformation matrix of B. The transformation consists following set of operations :

- Rotation by 90° about Z_0
- Translation by (45, – 40, 0)

$$^0H_B = H_{trans}(45, -40, 0)\, R(Z_0, 90°)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 45 \\ 0 & 1 & 0 & -40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 45 \\ 1 & 0 & 0 & -40 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*iii*) Homogeneous transformation of matrix C transformation consists of following sequence of operations :

- Rotation of 90° about Z_0
- Rotation of (– 90°) about X_0
- Translation by (– 40, 20, 50)

$${}^0H_C = H_{trans}(-40, 20, 50)\, R(Z_0, 90°)\, R(X_0, -90°)\, I_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & -40 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -40 \\ 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*iv*) Homogeneous transformation matrix for D Transformation involves following sequence of operations :

- Rotation by (– 90°) about Z_0
- Rotation by 90° about Y_0
- Translation of (0, 0, 50)

$${}^0H_D = H_{trans}(0, 0, 50)\, R(Z_0, -90°)\, R(Y_0, 90°)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogeneous transformation A from w.r.t. C frame

$${}^CH_A = [{}^CH_0]\,[{}^0H_A] = [{}^0H_C]^{-1}\,[{}^0H_A]$$

$$= \begin{bmatrix} 0 & 1 & 0 & -20 \\ 0 & 0 & -1 & 50 \\ -1 & 0 & 0 & -40 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 45 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & +25 \\ 0 & 1 & 0 & 30 \\ 1 & 0 & 0 & -40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 3.19. *A six joint robotic manipulator equipped with a digital TV camera is capable of continuously monitoring the position and orientation of an object. The position and orientation of the object with respect to the camera is expressed by a matrix $[T_1]$, the origin of the robot's base co-ordinate with respect to the camera is given by $[T_2]$, and the position and orientation of the gripper with respect to the base co-ordinate frame is given by $[T_3]$.*

$$[T_1] = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [T_2] = \begin{bmatrix} 1 & 0 & 0 & -20 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } [T_3] = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine

(*i*) *the position and orientation of the object with respect to the base co-ordinate.*

(*ii*) *the position and orientation of the object with respect to gripper.*

(VTU Jan.Feb. 2004 ; VTU May.June 2004)

Sol. Given :

$$[T_1] = {}^{camera}T_{object}$$
$$[T_2] = {}^{camera}T_{base}$$
$$[T_3] = {}^{base}T_{gripper}$$

(*i*) The position and orientation of object with respect to base co-ordinates, $[{}^{base}T_{object}]$

By chain product rule,

$${}^{base}T_{object} = {}^{base}T_{camera} \cdot {}^{camera}T_{object} = [T_2]^{-1}.[T_1]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 25 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Ans. The position vector = $[20, 4, 2]^T$

The orientation matrix = $[n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(*ii*) To determine the position and orientation of the object with respect to gripper, by chain product rule,

$${}^{gripper}T_{object} = {}^{gripper}T_{base} \cdot {}^{base}T_{object} = [T_3]^{-1}([T_2]^{-1}[T_1])$$

$$= \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 20 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{gripper}T_{object} = \begin{bmatrix} 0 & 1 & 0 & (-8+20) \\ -1 & 0 & 0 & (4-6) \\ 0 & 0 & 1 & (-6+2) \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Ans. The position vector = $[12, -2, -4]^T$

The orientation vectors, $[n, s, a] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

3.13 MANIPULATOR PARAMETERS

A robot manipulator is a chain of rigid bodies, called links, connected in sequence by joints, known as lower pair joints. The links remain in contact at the joints with two surfaces sliding over one another relatively. There are totally six possible lower pair joints : prismatic (sliding) joint, revolute (rotary) joint, cylindrical, screw, spherical and planar joints. The robot manipulators are generally, designed with prismatic or/and revolute joints. In a serial open loop formation each link forms connection, at the most, with two other links. Each pair of a link and a joint contributes single degree of freedom. 'N' numbers of pairs provide 'N' degrees of freedom for a manipulator. Link 1 forms a joint '0' with the base which establishes an innertial co-ordinate frame for a dynamic system anylysis of a industrial robot. The last link at its free end accomodates a tool or a gripper. Both the base and the gripper are not considered as the part of a robotic manipulator.

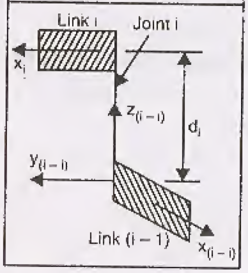
In general the link '*k*' gets connected at the two ends with link $(k-1)$ and link $(k+1)$, forming two joints at the ends of connections. The link is characterised by the (*i*) distance (d_i) and (*ii*) the angle (θ_i), between the adjacent links. The joint is featured by (*a*) length (q_k) and (*b*) the twist angle (α_k) of the link *k*. The manipulator parameters determine structure and relative position of links in the arm.

Table 3.6. Kinematic Parameters of Joints and Links

Kinematic Parameters of Manipulators

- Joint Parameters
  - Distance (d)
  - Angle (θ)
- Link Parameters
  - Length (a)
  - Twist angle (α)

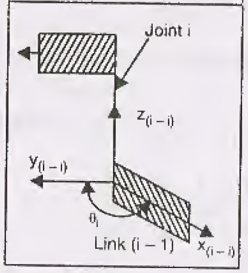
Distance (d)



Definition

The distance measured along joint axis between the normals, specify the relative position between the two connected variable for links a prismatic joint.

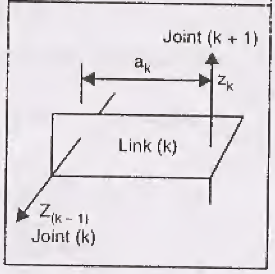
Angle (θ)



Definition

The angle measured between the normals in a plane perpendicular to the joint axis, specify relative orientation between adjacent links. Variable for a revolute joint.

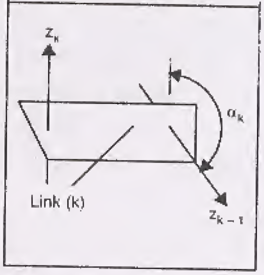
Length (a)



Definition

The shortest distance measured along the common normal between successive joint axes. Fixed for a given link design.

Twist angle (α)



Definition

Angle between the joint axes measured in a plane normal to a_k. Fixed for a particular link design.

Uses (Joint Parameters)

- Determine the position and orientation relative to the neighbouring link.
- Determine the kinematic configuration of each link in the manipulator.

Uses (Link Parameters)

- Determine the structure of a link between successive joints.
- Determine the kinematic configuration of each joint in the manipulator.

3.14 THE DENAVIT-HARTENBERG (D-H) REPRESENTATION

In an open kinematic manipulator chain of links and joints, each link (at joints) is assigned a right handed orthonormal co-ordinate frame and the manipulater parameters through a systematic procedure proposed by Denavit and Hartenberg in the year 1955. This procedure of assigning co-ordinate frame to the links helps in arriving at 4 × 4 homogeneous transformation matrix at each joint with respect to the previous joint or frame leading to the formulation of the kinematic arm equation.

D-H algorithm is a two pass procedure concept. The first pass is responsible for the assignment of frames starting from base to the distal end and the second pass computes the kinematic link-joint parameters.

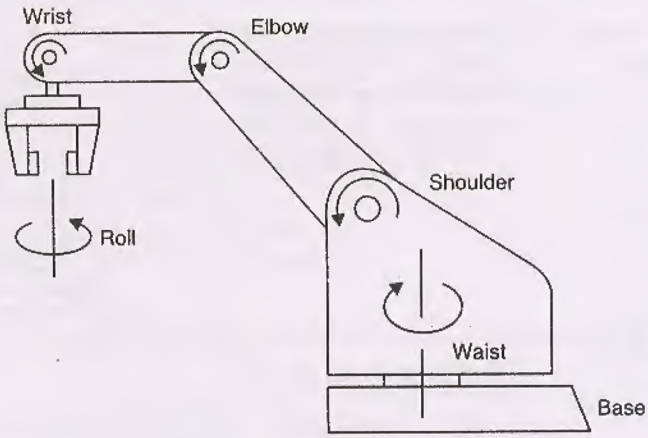
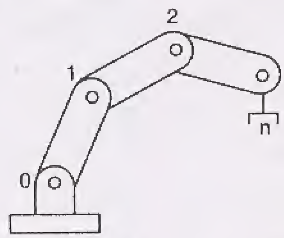
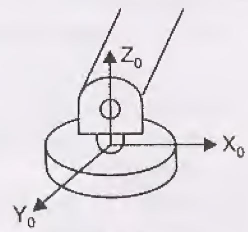
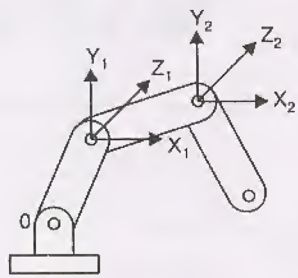


Fig. 3.18 A Robot Schematic.

Denavit-Hartenberg Algorithm

Table 3.7. (I) Frame Pass

Step	*Assignment*	*Where to*	*Illustration (Fig. 3.19)*
1.	Number the joints from 0 to n.	Start with the base and end with the tool taking yaw, pitch and roll in order.	
2.	Assign right hand ortho-normal co-ordinate frame A_0. Z_0 should align with the axis of joint '0'.	To the robot base	
3.	Assign Z_i with the other joints.	Along the axis of joints $(i + 1)$	

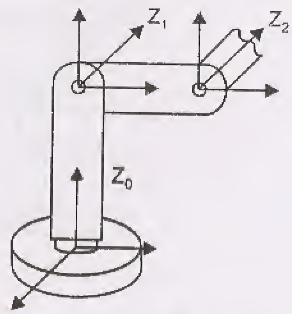
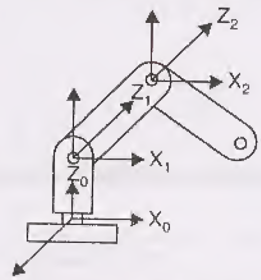
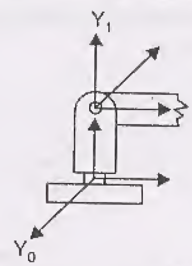
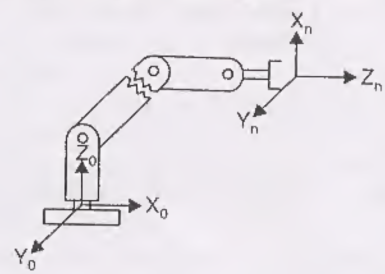
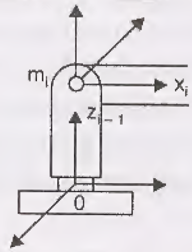
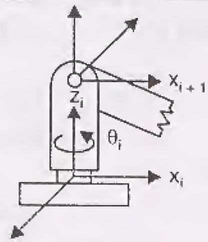
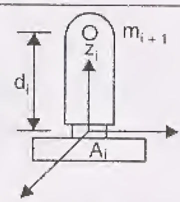
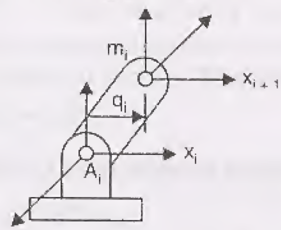
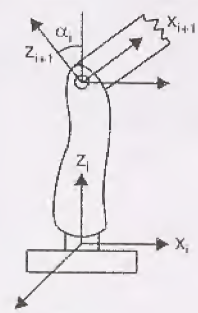
4.	Locate the origin of A_i with the joints.	At the intersection of Z_i and Z_{i-1}. If they do not intersect use the meet point of Z_i and the common normal between Z_i and Z_{i-1}.	
5.	• Selection of X_i. • If Z_i and Z_{i-1} are parallel,	Orthogonal to Z_i and Z_{i-1} • point x_i away from Z_{i-1}.	
6.	Assign y-co-ordinate axis as y_i,	So as to form ortho-normal co-ordinate frame with right hand rule	
7.	Set $i = i + 1$ and continue	If $i < n$, go to step 3.	
8.	Select origin of A_n • Align Z_n with tool approach • Align Y_n with the sliding vector • Align X_n with the normal vector + set $i = 1$	At the tool tip or end-effector tip.	

Table 3.8. The Parameter Pass

Step	*Assignment*	*Where to*	*Illustration (Fig. 3.19)*
1.	Locating point m_i If do not intersect	At the intersection of X_i and Z_{i-1} axis. Use common normal between X_i and Z_{i-1}.	
2.	Compute θ_i–the angle of rotation of joint i	Rotation of X_i into X_{i+1} measured about Z_i.	
3.	Compute d_i-distance between the joints	From frame origin A_i to point m_{i+1} measured along Z_i	
4.	Compute a_i-link distance,	From point m_i to frame origin A_i measured along X_i	
5.	Compute α_i - the twist angle of rotation of link	Angle is measured between Z_i and Z_{i+1} about X_{i+1}.	
6.	Set $i = i + 1$	if $i \leq n$, go to step 1.	

3.15 ARM MATRIX

After assigning co-ordinate frames to all the links, according to D-H representation, it is possible to establish the relation between successive frames i and $(i + 1)$ by the following rotations and translations in sequence.

- Rotation about Z_i by an angle θ_i
- Translate along Z_i by a distance d_i
- Translate along rotated $X_i = X_{i+1}$ through a length a_i
- Rotation about X_i by twist angle α_i

where $i = 0, 1, 2, \ldots n$.

This may be expressed as a product of four homogeneous transformations relating co-ordinate frame of $(i + 1)$ link to that of link (i). This relation is known as Arm matrix or A matrix.

$$
\begin{aligned}
{}^{i}A_{i+1} &= H(z, \theta).H_{trans}(0, 0, d).H_{trans}(a, 0, 0)\, H(x, \alpha) \\
&= H(z, \theta).H_{trans}(a, 0, d)\, H(x, \alpha) \qquad \ldots(3.26)
\end{aligned}
$$

Hence

$$
{}^{i}A_{i+1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos\theta & -\sin\theta.\cos\alpha & \sin\theta.\sin\alpha & a.\cos\theta \\ \sin\theta & \cos\theta.\cos\alpha & -\cos\theta.\sin\alpha & a.\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \ldots(3.27)
$$

For a prismatic joint $a = 0$.

The co-ordinate frame at the end of the manipulator is related to the base reference frame by the 'H' matrix in terms of A matrices, as below.

$$
{}^{0}H_6 = {}^{0}A_1.\,{}^{1}A_2.\,{}^{2}A_3.\,{}^{3}A_4.\,{}^{4}A_5.\,{}^{5}A_6 \qquad \ldots(3.28)
$$

• Transform Graph and Composite Matrix

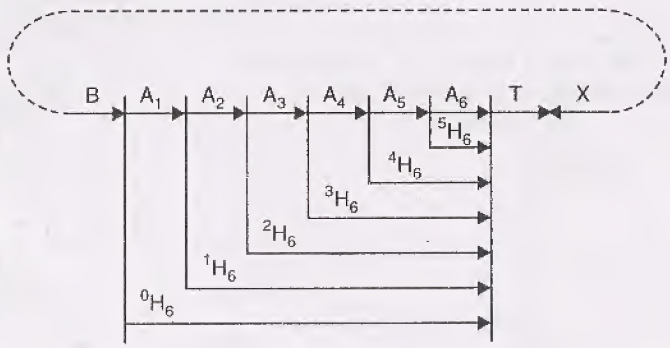


Fig. 3.20

where B ⇒ Transformation relating manipulator with the base reference frame.

T ⇒ Tool attachment description.

X ⇒ Position and orientation of tool tip with the base reference frame.

then $$X = B\,{}^0H_6E \qquad ...(3.29)$$

From the transform graph Fig. 3.20

$${}^0H_6 = B^{-1} \times E^{-1} \qquad ...(3.30)$$

3.16 KINEMATIC (ARM) EQUATIONS

• Stanford Manipulator

The schematic diagram and the co-ordinate frame diagram of a Stanford manipulator is as shown in Fig. 3.21(a) and Fig. 3.21(b) respectively. The following table gives the link co-ordinate parameters for a Stanford Robot Arm.

Table 3.9. Link Parameters

Joint 'i'	θ_i	d_i	α_i	α_i	$\cos\alpha_i$	$\sin\alpha_i$
0	θ_0	d_0	– 90°	0	0	– 1
1	θ_1	d_1	90°	0	0	+ 1
2	θ_2	d_2	0	0	1	0
3	θ_3	0	– 90°	0	0	– 1
4	θ_4	0	90°	0	0	+ 1
5	θ_5	d_5	0	0	1	0

To cut the length of the equations, the sine and the cosine of the angle θ are abbreviated in the following manner.

$$\sin\theta_i = S_i \qquad \cos\theta_i = C_i$$

$$\sin(\theta_i + \theta_j) = S_{ij} \qquad \cos(\theta_i + \theta_j) = C_{ij}$$

where $i, j = 0, 1, 2, 3, 4, 5$.

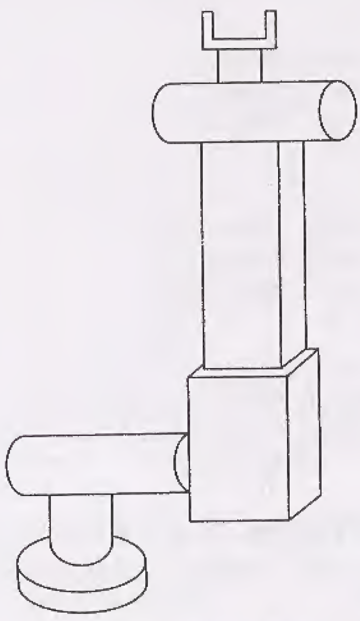


Fig. 3.21(a) Stanford Robot.

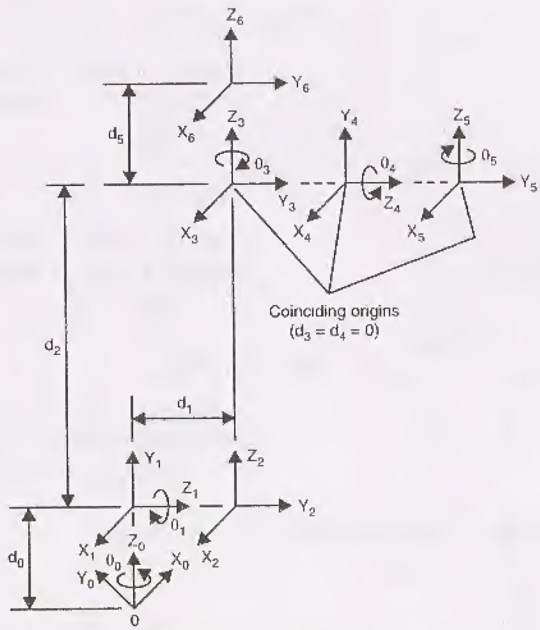


Fig. 3.21(b) Link Co-ordinate System.

The arm transformation matrices for Stanford manipulator are as follows [using equation 3.27]

$$^0A_1 = \begin{bmatrix} c_0 & 0 & -s_0 & 0 \\ s_0 & 0 & c_0 & 0 \\ 0 & -1 & 0 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1A_2 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 01 \end{bmatrix} \qquad ^3A_4 = \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4A_5 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^5A_6 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 01 \end{bmatrix}$$

- The kinematic Equation

$$^0H_6 = {}^0A_1 . {}^1A_2 . {}^2A_3 . {}^3A_4 . {}^4A_5 . {}^5A_6 .$$

$$^5H_6 = {}^5A_6 .$$

$$= \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 01 \end{bmatrix}$$

$$^4H_6 = {}^4A_5 . {}^5A_6 .$$

$$= \begin{bmatrix} c_4c_5 & -c_4s_5 & s_4 & d_5s_4 \\ s_4c_5 & -s_4s_5 & -c_4 & -d_5c_4 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3H_6 = {}^3A_4 . {}^4H_6 .$$

$$= \begin{bmatrix} c_3c_4c_5 - s_3s_5 & -c_3c_4s_5 - s_3c_5 & c_3s_4 & d_5c_3s_4 \\ s_3s_4c_5 + c_3s_5 & -s_3c_4s_5 + c_3c_5 & s_3s_4 & d_5s_3s_4 \\ -s_4c_5 & s_4s_5 & c_4 & d_5c_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2H_6 = {}^2A_3 \, {}^3H_6$$

$$= \begin{bmatrix} c_3c_4c_5 - s_3s_5 & -c_3c_4s_5 - s_3c_5 & c_3d_4 & d_5c_3s_4 \\ s_3c_4c_5 + c_3s_5 & -s_3c_4s_5 + c_3c_5 & s_3s_4 & d_5s_3s_4 \\ -s_4c_5 & s_4s_5 & c_4 & d_5c_4 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1H_6 = {}^1A_2 . {}^2H_6 .$$

$$= \begin{bmatrix} c_1(c_3c_4c_5 - s_3s_5) - s_1s_4c_5 & -c_1(c_3c_4s_5 + s_3c_5) + s_1s_4s_5 & c_1c_3s_4 + s_1c_4 & d_5.c_1c_3s_4 + s_1(d_5c_4 + d_2) \\ s_1(c_3c_4c_5 - s_3s_5) + c_1s_4c_5 & -s_1(c_3c_4s_5 + s_3c_5) - c_1s_4s_5 & s_1c_3s_4 - c_1c_4 & s_1d_5c_3s_4 - d_5c_1c_4 - d_2c_1 \\ +(s_3c_4c_5 + c_3s_5) & -s_3c_4s_5 + c_3c_5 & s_3s_4 & d_5c_4 + d_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $${}^0H_6 = {}^0A_1.{}^1H_6. = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ...(3.31)$$

The kinematic equations are

$$\begin{aligned}
n_x &= c_0c_1(c_3c_4c_5 - s_3s_5) - c_0s_1s_4s_5 - s_0(s_3c_4c_5 + c_3s_5) \\
s_x &= -c_0c_1(c_3c_4s_5 + s_3c_5) + c_0s_1s_4s_5 + s_0(s_3c_4s_5 - c_3c_5) \\
a_x &= c_0(c_1c_3s_4 + s_1c_4) - s_0s_3s_4 \\
n_y &= s_0c_1(c_3c_4c_5 - s_3s_5) - s_0s_1s_4c_5 + c_0(s_3c_4c_5 + c_3s_5) \\
s_y &= -s_0c_1(c_3c_4s_5 + s_3c_5) + s_0s_1s_4s_5 - c_0(s_3c_4s_5 - c_3c_5) \\
a_y &= s_0(c_1c_3s_4 + s_1c_4) + c_0s_3s_4 \\
n_z &= -[s_1(c_3c_4c_5 - s_3s_5) + c_1s_4c_5] \\
s_z &= s_1(c_3c_4s_5 + s_3c_5) + c_1s_4s_5 \\
a_z &= -s_1c_3s_4 + c_1c_4 \\
p_x &= d_5c_0c_1c_3s_4 + c_0s_1(d_5c_4 + d_2) - s_0(d_5c_4 + d_2 + d_2) \\
p_y &= d_5s_0c_1c_3s_4 + s_0s_1(d_5c_4 + d_2) + c_0(d_5c_4 + d_2 + d_1) \\
p_z &= -s_1d_5c_3s_4 + d_5c_1c_4 - d_2c_1 + d_0) \quad ...(3.32)
\end{aligned}$$

Table 3.10. Kinematic Parameters of SCARA

Joint i	θ_i	d_i	α_i	a_i
0	θ_0	d_0	π	a_0
1	θ_1	0	0°	a_1
2	0	d_2	0°	0
3	θ_3	d_3	0°	0

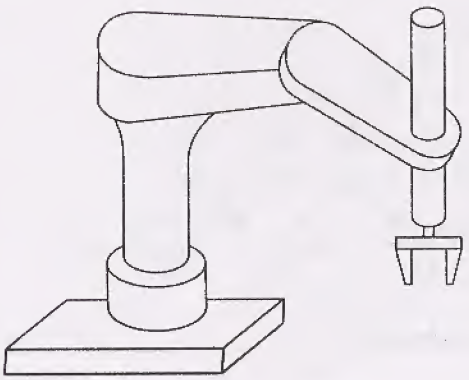


Fig. 3.22 (*a*) SCARA Robot.

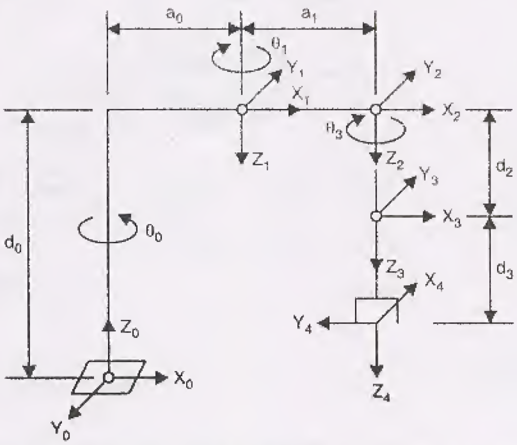


Fig. 3.22 (*b*) D-H Representation of SCARA.

Table 3.11. Kinematic Parameters of Elbow Manipulator

Joint i	θ_i	d_i	α_i	a_i
1	θ_1	0	90°	0
2	θ_2	0	0°	a_2
3	θ_3	0	0°	a_3
4	θ_4	0	– 90°	a_4
5	θ_5	0	90°	0
6	θ_6	0	0°	0

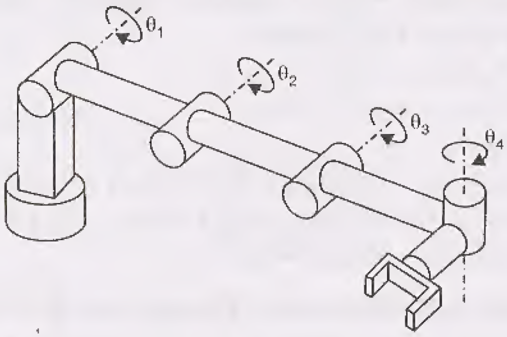


Fig. 3.23 (*a*) Schematic of Elbow Manipulator.

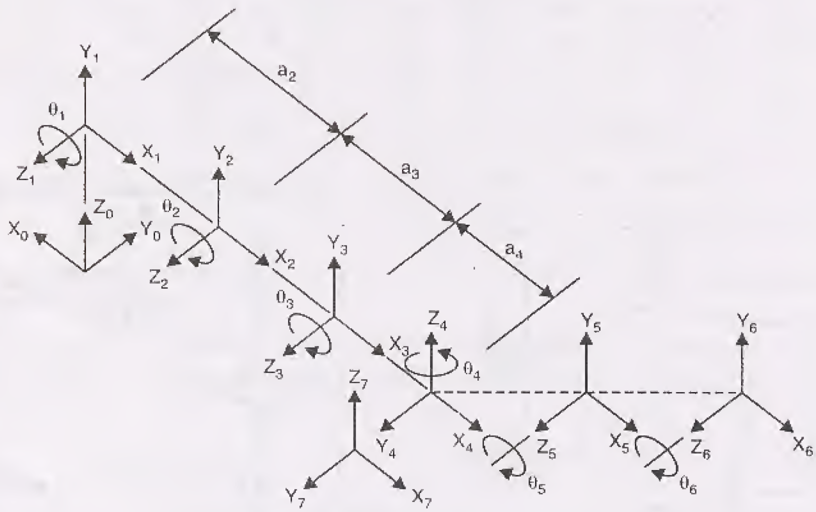


Fig. 3.23 (*b*) D-H Representation.

3.17 INVERSE KINEMATIC PROBLEM

Given the homogeneous transformation of the arm end with respect to the base reference co-ordinate system, the procedure of finding the rotation angles is known as inverse kinematic solution. In this section, the technique is demonstrated on the system II Eulerian angles.

There are three possible methods to inverse kinematic problem. The system II Euler angles resultant rotation matrix is given by the equation (3.17).

$$R(\phi, \theta, \psi) = R(z, \phi).\ R(B, \theta).\ R(C, \psi)$$

$$= \begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi s\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi s\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} \qquad ...(3.33a)$$

• *Method I*

Given matrix relating the manipulator end with the base is

$$R(\phi, \theta, \psi) = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix} \qquad ...(3.33b)$$

Comparing (3.33*a*) with (3.33*b*)

$$c\theta = a_z$$

$$\theta = \cos^{-1}(a_z) \qquad ...(3.34)$$

$$-s\theta c\psi = n_z$$

$$\psi = \cos^{-1}\left(\frac{n_z}{-s\theta}\right) \qquad ...(3.35)$$

and

$$c\phi s\theta = a_x$$

$$\phi = \cos^{-1}\left(\frac{a_x}{s\theta}\right) \qquad ...(3.36)$$

The behaviour of the arc cosine function is inconsistent as $\cos(\theta) = \cos(-\theta)$.

The matrix containing sine function is ill-conditioned as $\sin(\theta)$ approaches zero, as $\theta \to 0$ or $\pm 180°$.

The more consistent approaches are discussed in the following two methods.

• *Method II*

Pre-multiplication Technique

$$R(z, \phi)^{-1}\ H = R(B, \theta).\ R(C, \psi)$$

$$\begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n_x c\phi + n_y s\phi & s_x c\phi + s_y s\phi & a_x c\phi + a_y s\phi \\ -n_x s\phi + n_y c\phi & -s_x s\phi + s_y c\phi & -a_x s\phi + a_y c\phi \\ n_z & s_z & a_z \end{bmatrix} = \begin{bmatrix} c\theta c\psi & -c\theta s\psi & s\theta \\ s\psi & c\psi & 0 \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} \qquad ...(3.37)$$

Equating (2, 3) elements on RHS and LHS of (3.37).

$$-a_x s\phi + a_y c\phi = 0$$

and

$$\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) \qquad ...(3.38a)$$

Equating (2, 1) and (2, 2) elements on both sides of equation (3.37)

$$-n_x s\phi + n_y c\phi = s\psi$$

$$-s_x S\phi + s_y C\phi = C\psi$$

the solution for ψ is

$$\psi = \tan^{-1}\left[\frac{S\psi}{C\psi}\right] = \tan^{-1}\left[\frac{(-n_x S\phi + nyC\phi)}{(-s_x S\phi + s_y C\phi)}\right] \qquad ....(3.38b)$$

Equating (1, 3) element on both sides of (3.37)

$$S\theta = a_x\ C\phi + a_y S\phi$$

Equating (3, 3) element on both sides of (3.37)

$$C\theta = a_z$$

the solution for θ

$$\theta = \tan^{-1}\left[\frac{a_x C\phi + a_y S\phi}{a_z}\right] \qquad ...(3.39)$$

• *Method III*

Post-multiplication technique

H.R(c, ψ)$^{-1}$ = R (z, ϕ) R(B, θ)

$$\begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}\begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$\begin{bmatrix} n_x c\psi - s_x s\psi & n_x s\psi + s_x c\psi & a_x \\ n_y c\psi - s_y s\psi & n_y s\psi + s_y c\psi & a_y \\ n_z c\psi - s_z s\psi & n_z s\psi + s_z c\psi & a_z \end{bmatrix} = \begin{bmatrix} c\phi c\theta & -s\phi & c\phi s\theta \\ s\phi c\theta & c\phi & s\phi s\theta \\ -s\theta & 0 & c\theta \end{bmatrix} \qquad ...(3.40)$$

Equating (3, 2) element on both sides of equation (3.40).

$$n_z s\psi + s_z c\psi = 0$$

the solution for ψ

$$\psi = \tan^{-1}\left[-\frac{s_z}{n_z}\right] \qquad ...(3.41)$$

Equating the element (3, 1)

$$-s\theta = n_z c\psi - s_z s\psi$$

Equating the element (3, 3)

$$c\theta = a_z$$

The solution of angle θ

$$\theta = \tan^{-1}\left[\frac{-n_z c\psi + s_z s\psi}{a_z}\right] \qquad ...(3.42)$$

Equating element (1, 2)

$$-s\phi = n_z s\psi + s_x c\psi$$

Equating element (2, 2)

$$c\phi = n_y s\psi + s_x c\psi$$

The solution to angle ϕ

$$\phi = \tan^{-1}\left[\frac{-(n_x s\psi + s_x c\psi)}{(n_y s\psi + s_x c\psi)}\right] \qquad ...(3.43)$$

Example 3.20. *A two degree of freedom robot manipulator is shown in Fig. 3.24. Given that the length of each link is 1 unit. Establish link co-ordinate frame and the kinematic parameters. Find $^0A_1$ and $^1A_2$. Arrive at the inverse kinematic solution to this problem.*

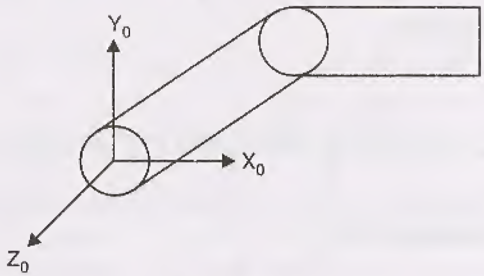


Fig. 3.24 (*a*)

Sol.

- Link co-ordinate frame.

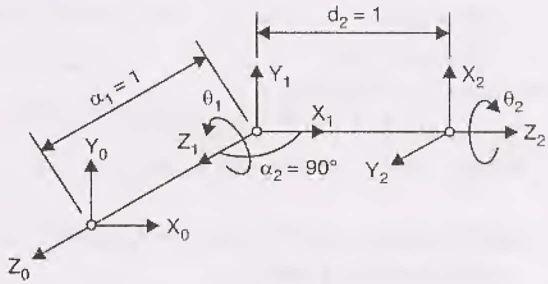


Fig. 3.24 (*b*)

- Kinematic (Link-joint) parameters

Table 3.12

Joint i	θ_i	d_i	α_i	a_i	*cos α*	*sin α*
1	θ_1	0	0	1	1	0
2	θ_2	1	90°	0	0	1

- Arm matrix $^0A_1$ and $^1A_2$.

Using the equation (3.27)

$$^iA_{i+1} = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & ac\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the values from table for kinematic parameters

$$^0A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & c_1 \\ s_1 & c_1 & 0 & s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^1A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_1 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Inverse kinematic solution.

Method I $\quad {}^1H_2 = {}^0A_1\, {}^1A_2$

$$ = \begin{bmatrix} c_1c_2 - s_1s_2 & 0 & c_1s_2 + s_1c_2 & c_1 \\ s_1c_2 + c_1s_2 & 0 & s_1s_2 - c_1c_2 & s_1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Comparing above elements with

$$ T = [n, s, a, p] = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Comparing the element (4, 1)

$$ \cos\theta_1 = p_x $$
$$ \theta_1 = \cos^{-1}(p_x) $$

Comparing the element (1, 1) and (2, 1)

$$ \cos\theta_1 \,.\, \sin\theta_2 - \sin\theta_1 \,.\, \sin\theta_2 = n_x $$
$$ \sin\theta_1 \,.\, \cos\theta_2 + \cos\theta_1 \sin\theta_2 = n_y $$

Simplifying

$$ -\sin^2\theta_1 \sin\theta_2 - \cos^2\theta_1 \sin\theta_2 = n_x \sin\theta_1 - n_y \cos\theta_1 $$
$$ -\sin\theta_2 (\sin^2\theta_1 + \cos^2\theta_1) $$

Hence $\quad \theta_2 = \sin^{-1}(n_y \cos\theta_1 - n_x \sin\theta_1)$

Method II $\quad T = {}^0A_1\, {}^1A_2$

$$ T.[{}^1A_2]^{-1} = {}^0A_1 $$

$$ \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_2 & -c_2 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & c_1 \\ s_1 & c_1 & 0 & s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \begin{bmatrix} n_xc_2 + a_xs_2 & n_xs_2 - a_xc_2 & s_x & p_x - a_x \\ n_yc_2 + a_ys_2 & n_ys_2 - a_yc_2 & s_y & p_y - a_y \\ n_zc_2 + a_zs_2 & n_zs_2 - a_zc_2 & s_z & p_z - a_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & c_1 \\ s_1 & c_1 & 0 & s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Comparing the element (3, 2)

$$ n_zs_2 - a_2c_2 = 0 $$
$$ \frac{s_2}{c_2} = \frac{a_z}{n_z} $$
$$ \theta_2 = \tan^{-1}\left(\frac{a_z}{n_z}\right) $$

Comparing the element (1, 4) and (2, 4)

$$\frac{s_1}{c_2} = \frac{p_y - a_y}{p_x - a_x} = \tan\theta_1$$

$$\theta_1 = \tan^{-1}\left[\frac{p_y - a_y}{p_x - a_x}\right].$$

3.18 GEOMETRIC APPROACH TO INVERSE KINEMATICS

To demonstrate the graphical approach to inverse kinematic problem, a two degree freedom robot manipulator is considered with its left and right configuration as dipicted in Fig. 3.25(*a*) and 3.26(*a*) respectively. Given a position, (p_x, p_y, p_z) of point p in the three dimensional space, the solution to the joint angle θ_1 is arrived at graphically for both left and right configurations, in terms of the co-ordinates p_x, p_y and p_z, as drawn in the Fig. 3.25(*b*) and 3.26(*b*) respectively.

- Left arm configuration :

From the Fig. 3.25(*b*) it is evident

$$\text{OM} = d_1$$

$$\left.\begin{aligned} \text{MP} &= \text{V} = \sqrt{p_x^2 + p_y^2 - d_1^2} \\ \text{OP} &= \text{R} = \sqrt{p_x^2 + p_y^2} \end{aligned}\right\} \quad \text{...(3.44}a\text{)}$$

$$(\theta_1)_\text{L} = \phi - \alpha$$

$$\left.\begin{aligned} \sin\phi &= \frac{p_y}{\text{R}}, \quad \cos\phi = \frac{p_x}{\text{R}} \\ \sin\alpha &= \frac{d_1}{\text{R}}, \quad \cos\alpha = \frac{\text{V}}{\text{R}} \end{aligned}\right\} \quad \text{...(3.44}b\text{)}$$

$$(\sin\theta_1)_\text{L} = \sin(\phi - \alpha)$$

$$= \sin\phi\cos\alpha - \cos\phi\sin\alpha$$

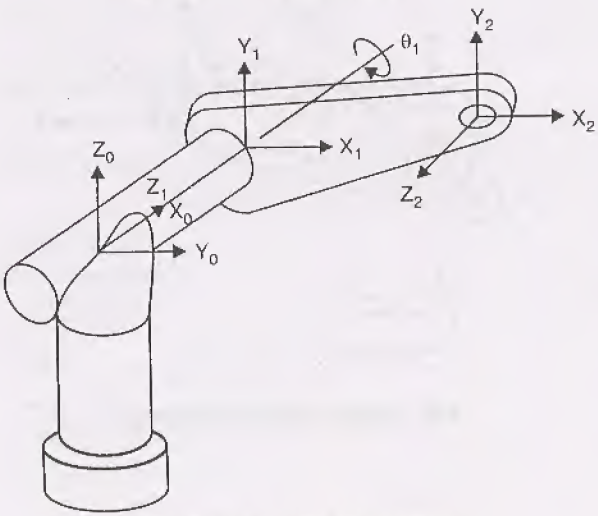


Fig. 3.25(*a*) Left Configuration.

Using the relation 3.44(*b*)

$$(\sin \theta_1)_L = \frac{p_y.V - p_x.d_1}{R^2} \qquad ...(3.45)$$

and

$$(\cos \theta_1)_L = \cos (\phi - \alpha)$$

$$= \cos \phi \cos \alpha + \sin \phi \sin \alpha$$

$$= \frac{p_x.V + p_y d_1}{R^2} \qquad ...(3.46)$$

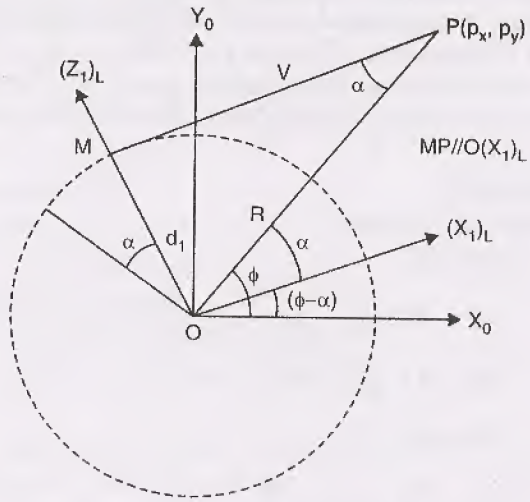


Fig. 3.25(*b*) Geometric Approach.

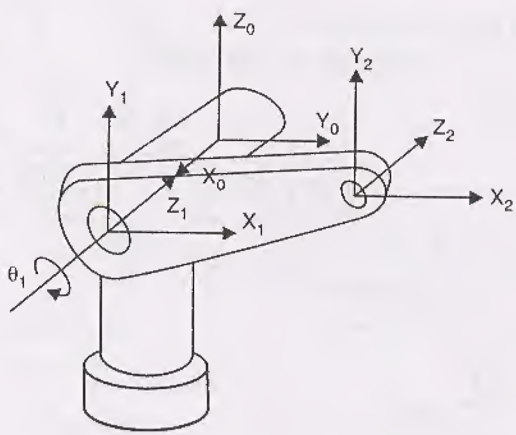


Fig. 3.26(*a*) Right Configuration.

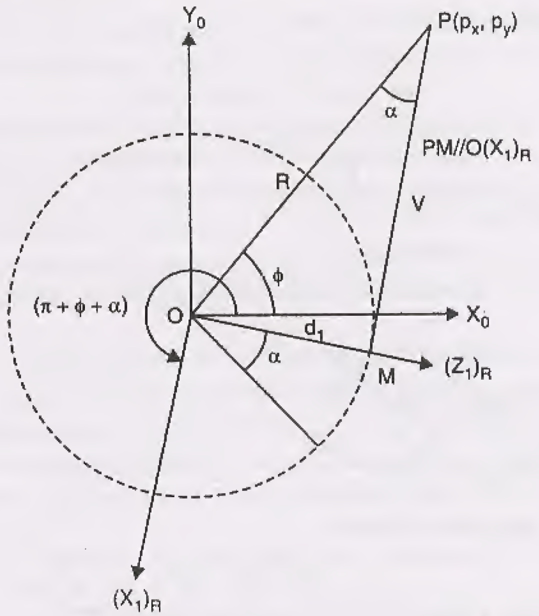


Fig. 3.26(*b*) Graphical Approach.

Using relation 3.44(*b*)

$$(\sin \theta_1)_L = \frac{p_y \,.\, V - p_x \,.\, d_1}{R^2} \qquad ...(3.47)$$

and

$$(\cos \theta_1)_L = \cos(\phi - \alpha)$$

$$= \cos\phi \cos\alpha + \sin\phi \sin\alpha$$

$$= \frac{p_x \,.\, V + p_y d_1}{R^2} \qquad ...(3.48)$$

From expressions (3.45) and (3.46)

$$(\theta_1)_L = \tan^{-1}\left[\frac{p_y . V - p_x d_1}{p_x . V + p_y d_1}\right]$$

Substituting the values for V and the expression for

$$(\theta_1)_L = \tan^{-1}\left[\frac{p_y\sqrt{p_x^2 + p_y^2 - d_1^2} - p_x d_1}{p_x\sqrt{p_x^2 + p_y^2 - d_1^2} + p_y d_1}\right] \qquad ...(3.49)$$

- Right Arm Configuration :

From Fig. 3.26(*b*) it is clear that

$$OM = d_1$$

$$MP = V = \sqrt{p_x^2 + p_y^2 - d_1^2}$$

$$OP = R = \sqrt{p_x^2 + p_y^2}$$

$$(\theta_1)_R = (\pi + \phi + \alpha)$$

$$\begin{aligned}(\sin\theta_1)_R &= \sin(\pi+\phi+\alpha)\\ &= -\sin(\phi+\alpha)\\ &= -[\sin\phi.\cos\alpha+\cos\phi\sin\alpha]\\ &= -\frac{[p_y.\mathrm{V}+p_x d_1]}{\mathrm{R}^2}\end{aligned} \qquad ...(3.50)$$

$$\begin{aligned}(\cos\theta_1)_R &= \cos(\pi+\phi+\alpha)\\ &= -\cos(\phi+\alpha)\\ &= -[\cos\phi.\cos\alpha-\sin\phi\sin\alpha]\\ &= -\frac{[p_x.\mathrm{V}-p_y d_1]}{\mathrm{R}^2}\end{aligned} \qquad ...(3.51)$$

From relations (3.48) and (3.49)

$$\tan(\theta_1)_R = \frac{p_y.\mathrm{V}-p_x d_1}{p_x \mathrm{V}-p_y.d_1}$$

Substituting from expression 3.44(*a*)

$$(\theta_1)_R = \tan^{-1}\left[\frac{p_y\sqrt{p_x^2+p_y^2-d_1^2}+p_x.d_1}{p_x\sqrt{p_x^2+p_y^2-d_1^2}-p_y.d_1}\right] \qquad ...(3.52)$$

The joint angle θ_1 is different for different (Left/Right) configurations.

EXERCISE

3.1. Using DH method and by symbolic sketch, write down the transformation matrices for each link and determine the position and orientation of end effector with respect to the base in a cartesian robot configuration. *(VTU–Jan./Feb. 2004)*

3.2. Write short notes on

(*a*) Direct and inverse kinematics

(*b*) DH convention *(VTU–Jan./Feb. 2004)*

3.3. Discuss about direct and inverse kinematics *(VTU–May./June. 2004)*

3.4. Draw any two Euler angle systems and show rotation and angles. *(VTU–May./June. 2004)*

3.5. Explain DH convention briefly. *(VTU–May./June. 2004)*

3.6. Using DH method and by neat sketch for an anthropomorphic arm, considering all link parameters write down the transformation matrices for each link and represent the position and orientation of E.E. frame with respect to base frame. *(VTU–Jan./Feb. 2004)*

3.7. Write short notes on inverse transforms. *(VTU–Jan./Feb. 2003)*

3.8. Arrive at the rotation matrices for the angle of rotations α, β and θ about x, y and z axes respectively.

3.9. Illustrate the geometric interpretation of the rotation transformations.

3.10. Derive the composite rotation matrix for the rotations about the cartesian axes. Write the rules applied in arriving at composite rotation matrix.

3.11. Discuss the operation of rotation about an arbitrary axis represented by a vector and derive the rotation matrix and give geometric interpretation.

3.12. Give Euler angles representation for the RPY system and derive the rotation matrix.

3.13. Explain the homogeneous transformation matrix and interprete the partitioning with application.

3.14. Explain the geometric interpretation of homogeneous transformations.

3.15. Explain the use of inverse transformation matrix in robotic application.

3.16. What is composite homogeneous transformation ? Explain the rules applied in the formation of the same.

3.17. Define and illustrate the link and joint parameters. Explain their uses.

3.18. Explain co-ordinate frame assignment of DH representation.

3.19. Explain the computation of kinematic parameters using DH algorithm.

3.20. List the steps involved in DH convention.

3.21. Derive the general arm matrix explaining the sequence of operations in arriving at it.

3.22. Derive the arm matrices for a cylindrical robot. Hence obtain the kinematic equations for the same.

3.23. Explain and derive inverse kinematic solution for the variables of a cylindrical robot.

3.24. Apply inverse kinematic solution to get Euler angles of RPY system of representation.

3.25. Get Euler angles for system I representation by applying inverse kinematic solution.

3.26. Explain the transform graph interpretation of homogeneous transformation.

3.27. Derive the kinematic equations for the SCARA robot giving co-ordinate frame diagram and the kinematic parameters.

3.28. Derive the kinematic equation for the elbow manipulator with co-ordinate frame diagram and kinematic parameters.

3.29. Explain the geometric solution of inverse kinematic with an example of two-degree system manipulator.

4 Robot Arm Statics and Dynamics

4.1 INTRODUCTION TO STATIC FORCES

For the robot to carryout the task the tool tip is required to exert the specific amount of force. This force can be resolved along three cartesian axes as f_x, f_y and f_z. Certain tasks cannot be performed only by the action of force. To produce rotation the torque is also required to be exerted which has the component along three axes of the frame, as τ_x, τ_y and τ_z.

Hence, the force vector can be expressed as

$$[F]^T = [f_x, f_y, f_z, \tau_x, \tau_y, \tau_z]$$

These forces can be transformed to give the joint forces and joint torques.

4.2 TRANSFORMATIONS

Let the co-ordinate frame A be described by the homogeneous transformation matrix

$$A = \begin{bmatrix} n_x & O_x & a_x & p_x \\ n_y & O_y & a_y & p_y \\ n_z & O_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If f represent components of force and m represent the torque vector, $m = [m_x, m_y, m_z]$ Then the vector F is given by (with respect to frame)

$$F = \begin{bmatrix} n.((f \times p) + m) \\ o.((f \times p) + m) \\ a.((f \times p) + m) \\ n.f \\ o.f \\ a.f \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_x \\ f_y \\ f_z \end{bmatrix}.$$

4.3 JACOBIAN AND FORCE VECTOR

As defined earlier, the generalized force vector is

$$F = [f_x, f_y, f_z, \tau_x, \tau_y, \tau_z]^T$$

The joint torque vector defined by

$$T = [m_1, m_2, m_3, m_4, m_5, m_6]^T$$

The differential vector resulting from force vector F be

$$\Delta X = [\delta_x, \delta_y, \delta_z, \delta\theta_x, \delta\theta_y, \delta\theta_z]^T$$

The vector of joint rotation be

$$\Delta\phi = [\delta\phi_1, \delta\phi_2, \delta\phi_3, \delta\phi_4, \delta\phi_5, \delta\phi_6]^T$$

From principle of virtual work

$$[F]^T \Delta X = [T]^T \Delta\phi$$

or

$$[F]^T \frac{\Delta X}{\Delta t} = [T]^T \frac{\Delta\phi}{\Delta t}.$$

The Jacobian is defined by

$$\frac{dX}{dt} = J\frac{d\phi}{dt}$$

Hence,

$$[F]^T J\frac{d\phi}{dt} = [T]^T \frac{d\phi}{dt}$$

By which

$$[T] = [J]^T[F]$$

4.4 FOR A TWO LINK MANIPULATOR

Consider a revolute joint two link manipulator with two links L_1, and L_2. Let the force vector at the tip of the manipulator be $f = f_x i + f_y j$

The transformation matrix at the tip with respect to base

$$A = \begin{bmatrix} C_{12} & -S_{12} & 0 & p_x \\ S_{12} & C_{12} & 0 & p_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\tau_1 = n\ .\ ((f \times p) + m)$$

$$= L_1S_{12}f_x + (L_1C_{12} + L_2)\,f_y$$

$$\tau_2 = L_2 f_y$$

But

$$[T] = [J]^T\ [F]$$

$$\begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} = \begin{bmatrix} L_1\, S_{12} & (L_2 + L_1C_{12}) \\ 0 & L_2 \end{bmatrix}\begin{bmatrix} f_x \\ f_y\end{bmatrix}.$$

Hence the Jacobian is

$$J = \begin{bmatrix} L_1S_{12} & 0 \\ L_2 + L_1C_{12} & L_2 \end{bmatrix}$$

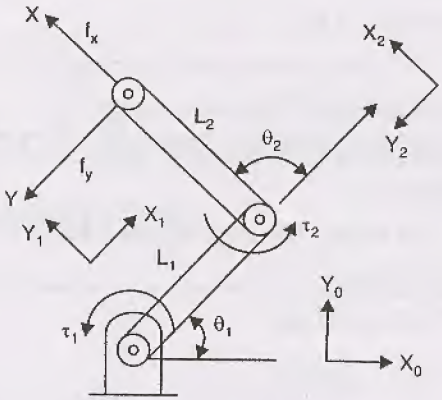


Fig. 4.1. Two Link Manipulator.

4.5 INTRODUCTION TO DYNAMICS

The study of dynamics of robot manipulator for the purposes of optimal performance and control has to be taken up through a systematic approach for solution to the complicated problem with several degree of freedom. As the number of links in the manipulator increases the degrees of freedom increase and hence complicate the dynamic analysis. The simplest and easier method to follow is using the Lagrangian Mechanics, which helps, in general, to obtain the dynamic equations. The dynamic equations establish relation between positions and forces/ torques. The time derivatives of positions *i.e.,* velocity and accelerations are also related to the force or torque through dynamic relations. Assuming the force and torques, it is possible to specify the motion characteristics of the manipulators through dynamic equations. On the other hand if the velocity and the accelerations are known, the dynamic equations provide the forces and torques to be applied at the joints for the stable control of the robot manipulator. The generation of dynamic equations and their solution for a complicated dynamic manipulator structure goes near impossibility, but the matrix presentation form provides useful informations for the control techniques.

A control system designer is particularly interested in obtaining.

- Approximate relation between torque and acceleration at a joint.
- Torque at one joint and accelerations at the other joints.
- The torque to be applied at a joint to overcome the effects of gravity.
- The torques depending upon velocities, being small compared to other system torques, are neglected (the symbolic solution to such a torque is also complicated as they involve too many variables). But in an high speed operation when the positional accuracy is of little importance, the velocity dependent torques play major role to play.

The simplified, symbolic dynamic equations are more useful than the numerical techniques. Even though numerical analysis provides solution to exact torques and forces as a function of positions, velocities and accelerations, they fail to provide sufficient insight into system analysis.

In this chapter we discuss the Lagrange-Euler formulation of dynamic equations based on generalised co-ordinates, generalised forces and two types of energies (K.E. and P.E.). The physical interpretations of the dynamic equations formed by L-E method are easy in application and analysis of the terms like manipulator link inertia, Coriolis component, centrifugal forces, gravity and friction.

The other approaches like Newton-Euler formulation, though complicated, are computationally efficient when the degree of freedom of the manipulator increases, needing a real time solution to the dynamic problems.

4.6 JOINT VELOCITIES

The velocity is the time derivative of the position of the joint in the robot manipulator. To understand the relation between the generalized co-ordinates and the link lengths, a two degree freedom system with a two link manipulator as shown in Fig. 4.1 (*a*) is assumed. The mass of both the links m_1 and m_2, lumped at the link ends and link lengths d_1 and d_2 respectively is hung above a gravity field of acceleration '*g*'.

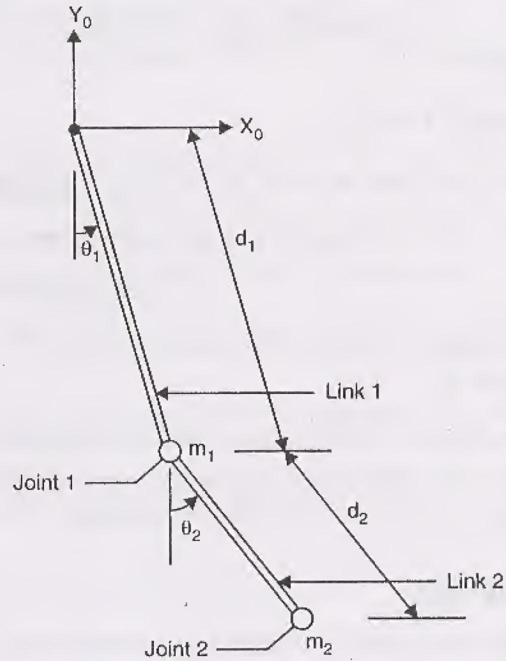


Fig. 4.1 (*a*) Two Link Manipulator.

Table 4.1

Link i	*Mass*	*Length*	*Generalized Co-ordinates*
Link 1	m_1	d_1	θ_1
Link 2	m_2	d_2	θ_2

By resolution at joint 1, we get cartesian co-ordinates

$$x_1 = d_1 \sin \theta_1$$
$$y_1 = -d_1 \cos \theta_1$$

By differentiating with respect to time

$$\dot{x}_1 = d_1 (\cos \theta_1)\, \dot{\theta}_1$$

$$\dot{y}_1 = +d_1 (\sin \theta_1)\, \dot{\theta}_1$$

Velocity,
$$V_1 = \sqrt{\dot{x}_1^{\,2} + \dot{y}_1^{\,2}}$$
$$= \sqrt{\left(d_1^2 \cos^2 \theta_1 + d_1^2 \sin^2 \theta_1\right)\dot{\theta}_1^{\,2}}$$
$$V_1 = d_1\, \dot{\theta}_1 \qquad \text{...(4.1)}$$

By resolution at joint 2, we get cartesian co-ordinates,
$$x_2 = d_1 \sin \theta_1 + d_2 \sin (\theta_1 + \theta_2)$$
$$y_2 = - d_1 \cos \theta_1 - d_2 \cos (\theta_1 + \theta_2)$$
and by differentiation w.r.t. time
$$\dot{x}_2 = d_1 \cos (\theta_1).\ \dot{\theta}_1 + d_2 \cos (\theta_1 + \theta_2) \left(\dot{\theta}_1 + \dot{\theta}_2\right)$$
$$\dot{y}_2 = d_1 \sin (\theta_1).\ \dot{\theta}_1 + d_2 \sin (\theta_1 + \theta_2) \left(\dot{\theta}_1 + \dot{\theta}_2\right)$$

The velocity of joint 2
$$V_2^{\,2} = (\dot{x}_2)^2 + (\dot{y}_2)^2$$
$$= \left(d_1\dot{\theta}_1\right)^2\left(\sin^2 \theta_1 + \cos^2 \theta_1\right) + \left[d_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\right]^2 \left[\sin^2 (\theta_1 + \theta_2)\right.$$
$$\left. + \cos^2[(\theta_1 + \theta_2)\right] + 2d_1 d_2 \cos (\theta_1) \cos (\theta_1 + \theta_2).\ \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2)$$
$$+ 2d_1 d_2.\ \sin (\theta_1) \sin(\theta_1 + \theta_2).\ \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2)$$

Simplifying, $V_2^{\,2} = (d_1\dot{\theta}_1)^2 + d_2^2\, (\dot{\theta}_1 + \dot{\theta}_2)^2 + 2d_1 d_2 \cos(\theta_2)(\dot{\theta}_1^{\,2} + \dot{\theta}_1\dot{\theta}_2)$

Hence, velocity of joint 2
$$V_2 = \sqrt{(d_1\dot{\theta}_1)^2 + d_2^2(\dot{\theta}_1 + \dot{\theta}_2) + 2d_1 d_2 \cos(\theta_2)(\dot{\theta}_1^{\,2} + \dot{\theta}_1\dot{\theta}_2)} \qquad \text{...(4.2)}$$

The mass multiplied with the velocity squared is proportional to the kinetic energy used in the formation of dynamic equation in the 'L-E' formulation.

4.7 THE KINETIC ENERGY

The mass 'm' moving with velocity 'v' has the kinetic energy given by
$$K = \frac{1}{2} m v^2 \qquad \text{...(4.3)}$$

K.E. for mass m_1
$$K_1 = \frac{1}{2}\, m_1 v_1^{\,2}$$

From equation (4.1)
$$K_1 = \frac{1}{2}\, m_1\, (d_1\dot{\theta}_1)^2 \qquad \text{...(4.4)}$$

K.E. for mass m_2
$$K_2 = \frac{1}{2}\, m_2 v_2^{\,2}$$

From equation (4.2)
$$= \frac{1}{2} m_2 \left[(d_1\dot{\theta}_1)^2 + d_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2d_1 d_2 \cos(\theta_2)(\dot{\theta}_1^{\,2} + \dot{\theta}_1\dot{\theta}_2)\right] \qquad \text{...(4.5)}$$

Total kinetic energy from equations (4.4) and (4.5)

$$\begin{aligned} K &= K_1 + K_2 \\ &= \frac{1}{2} m_1 d_1^2 (\dot{\theta}_1)^2 \; \frac{1}{2} m_2 d_1^2 (\dot{\theta}_1)^2 + \frac{1}{2} m_2 d_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \\ &\quad + m_2 d_1 d_2 \cos(\theta_2)(\dot{\theta}_1^{\,2} + \dot{\theta}_1 \dot{\theta}_2) \end{aligned} \qquad ...(4.6)$$

4.8 POTENTIAL ENERGY

The potential energy stored in a body is the function of mass, m, the gravitational acceleration, g and the height of the mass above the ground, h.

i.e., $$P = m.g.h.$$

In the two link manipulator shown in Fig. 4.1(a) the height of mass is expressed as the y-co-ordinate.

Hence P.E. of mass 'm_1'

$$P_1 = -\, m_1 g \,.\, d_1 \cos(\theta_1) \qquad ...(4.7)$$

and the potential energy of mass, m_2

$$P_2 = -\, m_2 g\, [d_1 \cos(\theta_1) + d_2 \cos(\theta_1 + \theta_2)] \qquad ...(4.8)$$

The total potential energy, from equations (4.7) and (4.8)

$$\begin{aligned} P &= P_1 + P_2 \\ &= -\,(m_1 + m_2)\, g\, d_1 \cos(\theta_1) - m_2 g\, d_2 \cos(\theta_1 + \theta_2) \end{aligned} \qquad ...(4.9)$$

The Lagrangian

The Lagrangian is defined as

$$L = (K - P) \qquad ...(4.10)$$

Using the equations (4.6) and (4.9)

$$\begin{aligned} L = \frac{1}{2}(m_1 + m_2)\, d_1^{\,2} \dot{\theta}_1^{\,2} + \frac{1}{2} m_2\, d_2^{\,2} (\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2\, d_1 d_2 \cos(\theta_2)(\dot{\theta}_1^{\,2} + \dot{\theta}_1 \dot{\theta}_2) \\ + (m_1 + m_2)\, g\, d_1 \cos(\theta_1) + m_2 g\, d_2 \cos(\theta_1 + \theta_2) \end{aligned} \qquad ...(4.11)$$

4.9 THE LAGRANGIAN EQUATION OF MOTION

While describing the position of an object or a point in the work-space in kinematics, in Chapter 3, we used orthogonal co-ordinates (x, y, z). In the Lagrangian formulation to specify the position of an object any variable can be used, uniquely. Such variables are known as 'generalized co-ordinates'.

The position vector in space is the function of generalized co-ordinates, $[q_1, q_2, q_3, q_4, ..., q_n]$ and time, t.

i.e., $$X = [(q_1, q_2, q_3, ..., q_n), t] \qquad ...(4.12)$$

From equation (4.3)

$$K = \frac{1}{2} m v^2 = \frac{1}{2} .m. (\dot{X})^2$$

and $$\frac{\partial K}{\partial \dot{q}} = \frac{1}{2} . m\,(2\dot{X}) . \frac{\partial \dot{X}}{\partial \dot{q}}$$

But
$$\frac{\partial \dot{X}}{\partial \dot{q}} = \frac{\partial X}{\partial q}$$

Hence
$$\frac{\partial K}{\partial \dot{q}} = m\dot{X}\left(\frac{\partial \dot{X}}{\partial \dot{q}}\right) = m\dot{X}\left(\frac{\partial X}{\partial q}\right) \quad ...(4.13)$$

Also
$$\frac{\partial K}{\partial q} = \frac{1}{2}.\, m\,(2\dot{X})\,\frac{\partial}{\partial q}(\dot{X})$$

$$= m(\dot{X})\,\frac{d}{dt}\left(\frac{\partial X}{\partial q}\right) \quad ...(4.14)$$

and
$$\frac{d}{dt}\left[\frac{\partial K}{\partial \dot{q}}\right] = \frac{d}{dt}\left[m\dot{X}\left(\frac{\partial X}{\partial q}\right)\right] \quad ...\text{[from equation 4.14]}$$

or
$$\frac{d}{dt}\left[\frac{\partial K}{\partial \dot{q}}\right] = m\,\ddot{X}\,\frac{\partial X}{\partial q} + m\dot{X}\,\frac{d}{dt}\left(\frac{\partial X}{\partial q}\right) \quad ...(4.15)$$

Define a function

$$Q = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q}$$

Using relations (4.14) and (4.15)

$$Q = m\ddot{X}\frac{\partial X}{\partial q} + m\dot{X}\,\frac{d}{dt}\left(\frac{\partial X}{\partial q}\right) - m\dot{X}.\frac{d}{dt}\left(\frac{\partial X}{\partial q}\right)$$

$$= m\ddot{X}\frac{\partial X}{\partial q} \quad ...(4.16)$$

where $\ddot{X}$ is the acceleration (second time derivative of the position vector X).

By Newton's equation of motion

$$Q = F.\frac{\partial X}{\partial q} = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q} \quad ...(4.17)$$

In the matrix form

$$Q_i = F_j^T\,\frac{\partial X_j}{\partial q_i} = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} \quad ...(4.18)$$

Lagrangian Function

Using equation (4.10)

$$L = K - P$$

$$Q_L = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} \quad ...(4.19)$$

$$Q_L = \frac{d}{dt}\left[\frac{\partial K}{\partial \dot{q}} - \frac{\partial P}{\partial \dot{q}}\right] - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q}$$

But
$$\frac{d}{dt}\left(\frac{\partial P}{\partial \dot{q}}\right) = 0$$

Hence,

$$Q_L = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q} \qquad ...(4.20)$$

In the matrix form

$$Q_{iL} = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} \qquad ...(4.21\,a)$$

and from equation (4.18)

$$F_{iL}^{T}\,\frac{\partial X_i}{\partial q_i} = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} \qquad ...(2.21\,b)$$

We can apply equations (4.21 *a*) and (4.21 *b*) to a system of rigid bodies with '*n*' degree of freedom system. Considering rigid bodies as a collection of infinitesimal mass. In such cases the kinetic energy is given as

$$K = \frac{1}{2} m_i \dot{X}_i^T\, \dot{X}_i + \frac{1}{2}\,\omega_i^T\, I\, \omega_i \qquad ...(4.22)$$

where $\dot{X}_i$ = translational velocity of the centre of mass

ω_i = angular velocity of body i

m_i = mass of body i

I = inertia centre of rigid bodies.

4.10 THE DYNAMICS EQUATION

(The dynamics equation for two link manipulator so as to obtain the dynamics equation for manipulator shown in Fig. 4.1, differentiate equation (4.10) which gives Lagrangian L.

According to equation (4.19)

$$Q_L = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \qquad ...(4.23)$$

Here, $q_i \equiv (\theta_1, \theta_2)$ and $\dot{q}_i \equiv (\dot{\theta}_1, \dot{\theta}_2)$.

L is given by equation (4.10)

Hence,

$$\frac{\partial L}{\partial \dot{\theta}_i} = \frac{1}{2}(m_1 + m_2)\, d_1^2\, (2\dot{\theta}_1) + \frac{1}{2} m_2 d_2^2\, (2\dot{\theta}_1) + \frac{1}{2} m_2 d_2^2\, (2\dot{\theta}_2)$$
$$+\, 2\, m_2 d_1 d_2 \cos(\theta_2)(\dot{\theta}_1) + m_2 d_1 d_2 \cos(\theta_2)\, \dot{\theta}_2 \qquad ...(4.24)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = (m_1 + m_2)\, d_1^2\, \ddot{\theta}_1 + m_2 {d_2}^2\, \ddot{\theta}_1 + m_2 {d_2}^2 \ddot{\theta}_2$$
$$+\, 2 m_2 d_1 d_2 \cos(\theta_2)\, \ddot{\theta}_1 + 2 m_2 d_1 d_2\, (\dot{\theta}_1)\,(\dot{\theta}_2)\,(-\sin\theta_2)$$
$$+\, m_2 d_1 d_2 \cos(\theta_2)\, \ddot{\theta}_2 + m_2\, d_1 d_2 (\dot{\theta}_2)^2\, (-\sin\theta_2) \qquad ...(4.25)$$

$$\frac{\partial L}{\partial \theta_1} = -\,(m_1 + m_2)\, g d_1 \sin(\theta_1) - m_2 g d_2 \sin(\theta_1 + \theta_2) \qquad ...(4.26)$$

Combining equations (4.25) and (4.26) and substituting into equation (4.23) we get the torque for joint 1.

$$T_1 = [(m_1 + m_2)\, d_1^2 + m_2\, d_2^2 + 2m_2 d_1 d_2 \cos(\theta_2)]\, \ddot{\theta}_1$$
$$+ [m_2 d_2^2 + m_2 d_1 d_2 \cos(\theta_2)]\, \ddot{\theta}_2$$
$$- 2m_2 d_1 d_2 \sin(\theta_2)\, \dot{\theta}_1\, \dot{\theta}_2 - m_2 d_1 d_2 \sin(\theta_2)\, (\dot{\theta}_2)^2$$
$$+ (m_1 + m_2)\, g d_1 \sin(\theta_1) + m_2 g d_2 \sin(\theta_1 + \theta_2) \quad ...(4.27)$$

Rewriting equation (4.27)

$$T_1 = D_{11}\, \ddot{\theta}_1 + D_{12}\, \ddot{\theta}_2 + D_{111}\, (\dot{\theta}_1)^2 + D_{122}\, (\dot{\theta}_2)^2$$
$$+ D_{112}\, \dot{\theta}_1 \dot{\theta}_2 + D_{121}\, \dot{\theta}_2 \dot{\theta}_1 + D_1 \quad ...(4.28\,a)$$

where D_{11} = **Effective inertia-1**

$$= (m_1 + m_2)\, d_1^2 + m_2\, d_2^2 + 2m_2 d_1 d_2 \cos(\theta_2) \quad ...(4.29)$$

D_{12} = **coupling inertia**

$$= m_2\, d_2^2 + m_2 d_1 d_2 \cos(\theta_2) \quad ...(4.30)$$

Centripetal Acceleration co-efficients

$$D_{111} = 0$$
$$D_{122} = - m_2 d_1 d_2 \sin(\theta_2) \quad ...(4.31)$$

- Coriolis acceleration co-efficients

$$D_{112} = - m_2 d_1 d_2 \sin(\theta_2) = D_{121}$$

- **Gravity Term**

$$D_1 = (m_1 + m_2)\, g d_1 \sin(\theta_1) + m_2\, g d_2 \sin(\theta_1 + \theta_2)$$

To determine the torque at joint 2

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2\, d_2^2\, \dot{\theta}_1 + m_2\, d_2^2\, \dot{\theta}_2 + m_2 d_1 d_2 \cos(\theta_2)\, \dot{\theta}_1 \quad ...(4.32)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2\, d_2^2\, \ddot{\theta}_1 + m_2\, d_2^2\, \ddot{\theta}_2 + m_2 d_1 d_2 \cos(\theta_2)\, \ddot{\theta}_1$$
$$- m_2 d_1 d_2 \sin(\theta_2)\, \dot{\theta}_1\, \dot{\theta}_2 \quad ...(4.33)$$

and

$$\frac{\partial L}{\partial \theta_2} = - m_2 g d_2 \sin(\theta_1 + \theta_2) \quad ...(4.34)$$

Combining equations (4.33) and (4.34) and substituting into equation (4.23), we arrive at the torque at joint 2.

$$T_2 = [m_2 d_2^2 + m_2 d_1 d_2 \cos(\theta_2)]\, \ddot{\theta}_1 + m_2\, d_2^2\, \ddot{\theta}_2$$
$$- 2m_2 d_1 d_2 \sin(\theta_2)\, \dot{\theta}_1\, \dot{\theta}_2 + m_2 d_1 d_2 \sin(\theta_2)\, \dot{\theta}_2\, \dot{\theta}_1 + m_2 g d_2 \sin(\theta_1 + \theta_2)$$

Rewriting T_2 in the form

$$T_2 = [D_{12}\, \ddot{\theta}_1 + D_{22}\, \ddot{\theta}_2] + [D_{211}\, \dot{\theta}_1^2 + D_{222}\, \dot{\theta}_2^2]$$
$$+ [D_{212}\, \dot{\theta}_1\, \dot{\theta}_2 + D_{221}\, \dot{\theta}_2\, \dot{\theta}_1] + [D_2] \quad ...(4.28\,b)$$

- **Effective Inertia**

$$D_{22} = m_2\, d_2^2$$

- **Coupling Inertia**

$$D_{12} = m_2 d_2^2 + m_2 d_1 d_2 \cos(\theta_2)$$

- **Centripetal Acceleration Co-efficients**

$$D_{211} = -2m_2 d_1 d_2 \sin\theta_2$$
$$D_{222} = 0$$

- **Coriolis Co-efficient**

$$D_{212} = D_{221} = m_2 d_1 d_2 \sin(\theta_2)$$

- **Gravity Term**

$$D_2 = m_2 g d_2 \sin(\theta_1 + \theta_2)$$

Condition 1 : The joint 2 fixed

When the joint '2' is fixed we have $\ddot{\theta}_2 = 0$

Hence

$$T_1 = D_{11}\ddot{\theta}_1 \text{ from equation (4.28 } a)$$

$$T_2 = D_{12}\ddot{\theta}_1 \text{ from equation (4.28 } b)$$

Condition 2 : Joint 2 is free.

When the joint 2 is free $\ddot{\theta}_2 \neq 0$, but $T_2 = 0$

The equation (4.28 *b*) takes the form

$$T_2 = D_{12}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 = 0$$

$$\ddot{\theta}_2 = -\frac{D_{12}\ddot{\theta}_1}{D_{22}} \qquad ...(4.35)$$

and

$$T_1 = \left[D_{11} - \frac{D_{12}^2}{D_{22}}\right]\ddot{\theta}_1 \qquad ...(4.36)$$

Table 4.2. Components and Co-efficients for T_1

Components	*Co-effts.*	*Variables*	*Expressions*
• Effective inertia	D_{11}	$\ddot{\theta}_1$	$(m_1 + m_2) d_1^2 + m_2 d_2^2 + 2m_2 d_2 d_1 \cos(\theta_2)$
• Coupling inertia	D_{12}	$\ddot{\theta}_2$	$m_2 d_2^2 + m_2 d_1 d_2 \cos(\theta_2)$.
• Centripetal acceleration	(1) D_{111}	$(\dot{\theta}_1)^2$	0
	(2) D_{122}	$(\dot{\theta}_2)^2$	$-m_2 d_1 d_2 \sin(\theta_2)$
• Coriolis components	D_{112} D_{121}	$(\dot{\theta}_1\dot{\theta}_2)$	$-m_2 d_1 d_2 \sin(\theta_2)$
• Gravity term	D_1	–	$(m_1 + m_2) g d_1 \sin(\theta_1) + m_2 g d_2 \sin(\theta_1 + \theta_2)$

Table 4.3. Components and Co-efficients for T_2

• Effective inertia	D_{22}	$\ddot{\theta}_2$	$m_2 d_2^{\,2}$
• Coupling inertia	D_{21}	$\ddot{\theta}_1$	$m_2 d_2^{\,2} + m_2 d_1 d_2 \cos(\theta_2)$
• Centripetal acceleration	D_{211}	$\dot{\theta}_1^{\,2}$	$-2m_2 d_1 d_2 \sin(\theta_2)$
co-efficient	D_{222}	$\dot{\theta}_2^{\,2}$	0
• Coriolis componets	D_{212}	$(\dot{\theta}_2, \dot{\theta}_1)$	$m_2 d_1 d_2 \sin(\theta_2)$
	D_{221}		
• Gravity term	D_2	–	$m_2 g d_2 \sin(\theta_1 + \theta_2)$

4.11 DYNAMIC EQUATION FOR A GENERAL MANIPULATOR

The derivation of dynamic equation for a '*n*' degree manipulator gets an insight from the two link manipulator dynamics. The various steps involved to arrive at the general equation are

- Computation of velocity of any point in any link.
- Computation of kinetic energy.
- Computation of potential energy.
- Formulation of Lagrangian.
- Differentiation of Langrangian to get dynamic equations.

Step 1: Velocity of a Point

Let r_i be a point or a joint in the link '*i*'.

H_i be the homogeneous transformation with respect to base frame.

The position vector r is defined as

$$r = H_i \, . \, r_i \qquad \text{...(4.37)}$$

The velocity is

$$\frac{dr}{dt} = \frac{\partial}{\partial t}[H_i \; r_i]$$

or

$$= \frac{\partial H_i}{\partial q_m} \frac{\partial q_m}{\partial t} \, . \, r_i$$

In the summation for different points

$$\frac{dr}{dt} = \sum_{m=1}^{i} \left(\frac{\partial H_i}{\partial q_m} \, . \, \dot{q}_m \right) r_i \qquad \text{...(4.38)}$$

Velocity squared is

$$\left(\frac{dr}{dt}\right)^2 = \left(\sum_{m=1}^{i} \frac{\partial H_i}{\partial q_m} \, . \, \dot{q}_m \; r_i \right) \left(\sum_{n=1}^{i} \frac{\partial H_i}{\partial q_n} \, . \, \dot{q}_n \, . \, r_i \right)^{T}$$

In the matrix form

$$\left(\frac{dr}{dt}\right)^2 = \sum_{m=1}^{i} \sum_{n=1}^{i} \frac{\partial H_i}{\partial q_m}, r_i \, r_i^{T} \, \frac{\partial H_i^{T}}{\partial q_n}, \dot{q}_m, \dot{q}_n \qquad \text{...(4.39)}$$

Step 2: Kinetic Energy

An element of mass dm locationed on link 'i' at a distance r_i has the kinetic energy

$$dk_i = \frac{1}{2}\left(\frac{dr}{dt}\right)^2 . dm$$

$$= \frac{1}{2}\left[\sum_{m=1}^{(i)}\sum_{n=1}^{(i)}\frac{\partial H_i}{\partial q_m} r_i\, r_i^T \frac{\partial H_i^T}{\partial q_n} . \dot{q}_m . \dot{q}_n\right] dm.$$

In the matrix form

$$dK_i = \frac{1}{2}\,\text{Trace}\left[\sum_{m=1}^{i}\sum_{n=1}^{i}\frac{\partial H_i}{\partial q_m} r_i\, dm\, r_i^T \frac{\partial H_i^T}{\partial q_n} . \dot{q}_m . \dot{q}_n\right]$$

The total kinetic energy of the link i

$$K_i \int_i dK_i = \frac{1}{2}\,\text{Trace}\left[\sum_{m=1}^{i}\sum_{n=1}^{i}\frac{\partial H_i}{\partial q_m}\int_i r_i\, dm\, r_i^T \frac{\partial H_i^T}{\partial q_n}\dot{q}_m\, \dot{q}_n\right] \qquad ...(4.40)$$

or

$$K = \sum_{i=1}^{6} k_i = \frac{1}{2}\sum_{1}^{6}\text{Trace}\left[\sum_{m=1}^{i}\sum_{n=1}^{i}\frac{\partial H_i}{\partial q_m} . J_i \frac{\partial H_i^T}{\partial q_n}\dot{q}_m\, \dot{q}_n\right] \qquad ...(4.41)$$

where J_i is pseudo inertia Matrix given by

$$J_i = \int_i r_i\, r_i^T\, dm.$$

$$= \begin{bmatrix} \int x_i^2\, dm & \int x_i y_i\, dm & \int x_i\, z_i\, dm & \int x_i\, dm \\ \int y_i\, x_i\, dm & \int y_i^2\, dm & \int y_i\, z_i\, dm & \int y_i\, dm \\ \int z_i\, x_i\, dm & \int z_i\, y_i\, dm & \int z_i^2\, dm & \int z_i\, dm \\ \int x_i\, dm & \int y_i\, dm & \int z_i\, dm & \int dm \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-I_{xx}+I_{yy}+I_{zz}}{2} & I_{xy} & I_{xz} & m_i\,\bar{x}_i \\ I_{xy} & \frac{I_{xx}-I_{yy}+I_{zz}}{2} & I_{yz} & m_i\,\bar{y}_i \\ I_{xz} & I_{yz} & \frac{I_{xx}+I_{yy}-I_{zz}}{2} & m_i\,\bar{z}_i \\ m_i\,\bar{x}_i & m_i\,\bar{y}_i & m_i\,\bar{z}_i & m_i \end{bmatrix} \qquad ...(4.42)$$

where

$$I_{xx} = \int (y^2 + z^2)\, dm$$

$$I_{yy} = \int (z^2 + x^2)\, dm$$

$$I_{zz} = \int (x^2 + y^2)\, dm$$

$$I_{xy} = \int xy\, dm$$

$$I_{yz} = \int yz\, dm$$

$$\mathrm{I}_{zx} = \int zx\, dm$$

$$m\bar{x} = \int x\, dm,\ \int y\, dm = m\bar{y},\ m\bar{z} = \int z\, dm \qquad ...(4.43)$$

The actuator at the joints contribute considerably to the kinetic energy. Hence the total kinetic energy with the addition of K. E. due to actuator inertia is given by

$$\text{K.E.} = \mathrm{K} = \frac{1}{2}\sum_{1}^{6}\sum_{m=1}^{i}\sum_{n=1}^{i} \text{Trace}\left(\frac{\partial \mathrm{H}_i}{\partial q_m}.\mathrm{J}_i\,\frac{\partial \mathrm{H}_i^{\mathrm{T}}}{\partial q_n}\right)\dot{q}_m\,\dot{q}_n + \frac{1}{2}\sum_{1}^{6}\mathrm{I}_{am}\,\dot{q}_m \qquad ...(4.44)$$

Step 3: Potential Energy

From the mechanics a mass, m located at a height, h from a datum has the potential energy given by

$$\mathrm{P} = mgh, \quad \text{where } g \text{ is a gravitational constant.}$$

If the position of the centre of mass of a body is represented by $\bar{r}$, then

$$\mathrm{P} = mg\bar{r}$$

For a link i $\bar{r} = \bar{r}_i$ and $\mathrm{P} = mg\bar{r}_i$

If $\bar{r}_i$ is related to base frame by a transformation.

Then $\bar{r}_i = \mathrm{H}_i\bar{r}_i$

So $\mathrm{P} = mg\,\mathrm{H}_i\bar{r}_i$

The P is pointing down-wards and Y-axis is pointing upwards, hence

$$\mathrm{P} = -\,mg\,\mathrm{H}_i\bar{r}_i$$

To express in the matrix form

$$\mathrm{P}_i = -\,m_i\,g^{\mathrm{T}}\,\mathrm{H}_i\,\bar{r}_i \qquad ...(4.45)$$

where $g^{\mathrm{T}} = [g_x, g_y, g_z, 0]$

The total potential energy of the manipulator

$$\mathrm{P} = -\sum_{1}^{6} m_i\,g^{\mathrm{T}}\,\mathrm{H}_i\,.\,\bar{r}_i \qquad ...(4.46)$$

Step 4: Formulation of Lagrangian

The Lagrangian is given by

$$\mathrm{L} = \mathrm{K} - \mathrm{P}.$$

i.e.,

$$\mathrm{L} = \frac{1}{2}\sum_{1}^{6}\sum_{m=1}^{i}\sum_{n=1}^{i}\text{Trace}\left(\frac{\partial \mathrm{H}_i}{\partial q_m}.\mathrm{J}_i.\frac{\partial \mathrm{H}_i^{\mathrm{T}}}{\partial q_n}\right) q_m\,q_n + \frac{1}{2}\sum_{i=1}^{6}\mathrm{I}_{ai}\,q_i^{\,2}$$

$$+\sum_{i=1}^{6} m_i\,g^{\mathrm{T}}\,\mathrm{H}_i\,\bar{r}_i \qquad ...(4.47)$$

Step 5: Lagrange-Euler Formulation of Dynamic Equation

i.e.,

$$\mathrm{F}_i = \sum_{j=1}^{6}\mathrm{D}_{ij}\,\ddot{q}_j + \mathrm{I}_{ai}\,\ddot{q}_i + \sum_{j=1}^{6}\sum_{k=1}^{6}\mathrm{D}_{ijk}\,\ddot{q}_j\,\ddot{q}_k + \mathrm{D}_i \qquad ...(4.48)$$

where $$D_{ij} = \sum_{max\, i,j}^{6} \text{Trace}\left(\frac{\partial H_P}{\partial q_j} . J_P . \frac{\partial H_P^T}{\partial q_i}\right) \quad \text{...(4.49)}$$

$$D_{ijk} = \sum_{max\, i,j,k}^{6} \text{Trace}\left(\frac{\partial^2 H_P}{\partial q_j\, \partial q_k} . J_P . \frac{\partial H_P^T}{\partial q_i}\right) \quad \text{...(4.50)}$$

$$D_i = \sum_{p=i}^{6} - m_P\, g^T \frac{\partial H_P}{\partial q_i} \bar{r}_P \quad \text{...(4.51)}$$

F_i is the joint force or torque given by the following L–E formulation

i.e., $$F_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}.$$

- **What do the co-efficients 'D' represent ?**

  D_{ii} = effective inertia at joint i.

  D_{ij} = coupling inertia between joints i and j.

  D_{ijj} = centripetal force at joint i due to velocity at joint j.

  D_{ijk} = Coriolis forces at joint i due to velocities at j and k joints.

  D_i = Gravitational force at joint i.

- **Importance of 'D' co-efficients**

The inertial terms given by D_{ii} and D_{ij}, and the gravity term due to D_i affect the servo-stability and the positioning accuracy useful in manipulator control.

The centripetal and the Coriolis forces obtained using D_{ijj} and D_{ijk} are of particular importance in the high speed operation of the manipulator.

The actuator inertia I_{ai} has the decreasing effect an effective inertia and the coupling inertia forces.

PROBLEMS

Example 4.1. *Find the pseudo-inertia matrix for a rectangular body of density $10 kg/m^3$ with respect to the co-ordinate system as shown in Fig. 4.2.*

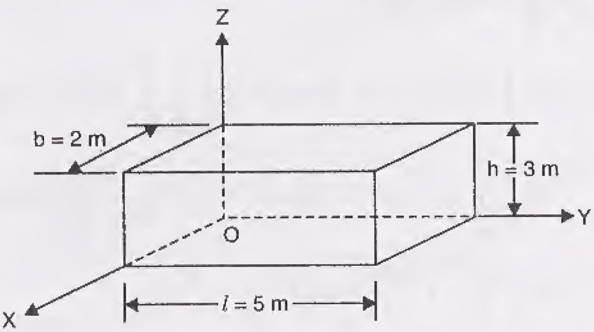


Fig. 4.2

Sol.

Data : Density, $\rho = 10\ kg/m^3$

length, $l = 5$ m

breadth, $b = 2$ m

height, $h = 3$ m.

Using the equation (4.42) the pseudo-inertia matrix is given by

$$J_i = \begin{bmatrix} -I_{xx} + I_{yy} + I_{zz} & I_{xy} & I_{xz} & m_i\bar{x}_i \\ I_{yx} & I_{xy} - I_{yy} + I_{zz} & I_{yz} & m_i\bar{y}_i \\ I_{zx} & I_{zy} & I_{xx} + I_{yy} - I_{zz} & m_i\bar{z}_i \\ m_i\bar{x}\ i & m_i\bar{y}_i & m_i\bar{z}_i & m_i \end{bmatrix}$$

To compute I_{xy}

$$I_{xy} = \int_0^h \int_0^l \int_0^b xy\, dm = \int_0^h \int_0^l \int_0^b xy\,(\rho\, dx\, dy\, dz)$$

$$= \rho \int_0^h \int_0^l \frac{b^2}{2} y\, dy\, dz = \rho \int_0^h \frac{b^2 l^2}{4} dz = \rho \frac{hb^2 l^2}{4}$$

$$I_{xy} = \rho V \frac{lb}{4} \quad \text{...(4.52)}$$

Substituting values for ρ, l, b, and h

$$I_{xy} = 10\,(2 \times 3 \times 5)\left(\frac{5 \times 2}{4}\right) = 750 \text{ kg. m}^2$$

Permuting the terms of equation (4.52)

$$I_{yz} = \rho V \left(\frac{lh}{4}\right)$$

$$= 10\,(2 \times 3 \times 5)\left(\frac{5 \times 3}{4}\right) = 1125 \text{ kg-m}^2.$$

$$I_{zx} = \rho V \left(\frac{bh}{4}\right) = 10\,(2 \times 3 \times 5)\left(\frac{3 \times 2}{4}\right) = 450 \text{ kg-m}^2.$$

Using relation (4.43)

$$I_{xx} = \int_V (y^2 + z^2)\, dm$$

$$= \rho \int (y^2 + z^2)\, dx.\, dy.\, dz = \rho \int_0^h \int_0^l \int_0^b (y^2 + z^2)\, dx.\, dy.\, dz.$$

$$= \rho \int_0^h \int_0^l (y^2 + z^2)\, b.\, dy.\, dz = \rho \int_0^h \left(\frac{l^3}{3} + \frac{lz^2}{3}\right) b.\, dz$$

$$= \rho \left[\frac{bhl^3}{3} + \frac{blh^3}{3}\right] = \rho V \left[\frac{l^2}{3} + \frac{h^2}{3}\right] \quad \text{...(4.53)}$$

Substituting for ρ, l, b and h

$$I_{xx} = 10\,(2 \times 3 \times 5)\left[\frac{25}{3} + \frac{9}{3}\right] = 3400 \text{ kg-m}^2.$$

Permuting the terms of equation (4.53)

$$I_{yy} = \rho V\left[\frac{b^2}{3} + \frac{h^2}{3}\right] = 300\left[\frac{4}{3} + \frac{9}{3}\right] = 1300 \text{ kg-m}^2.$$

$$I_{zz} = \rho V\left[\frac{b^2}{3} + \frac{l^2}{3}\right] = 300\left[\frac{4}{3} + \frac{25}{3}\right] = 2900 \text{ kg-m}^2.$$

and $\bar{x}_i = \frac{b}{2} = 1$, $\bar{y}_i = \frac{l}{2} = 2.5$, $\bar{z}_i = \frac{h}{2} = 1.5$

Hence,

$$m_i\bar{x}_i = \rho V\bar{x}_i = 10 \times 30 \times 1 = 300 \text{ kg-m}$$

$$m_i\bar{y}_i = \rho V\bar{y}_i = 300 \times 2.5 = 750 \text{ kg-m}$$

$$m_i\bar{z}_i = \rho V\bar{z}_i = 300 \times 1.5 = 450 \text{ kg-m}$$

and $m_i = \rho.V = 10\,(2 \times 3 \times 5) = 300$ kg.

$$J_i = \begin{bmatrix} 800 & 750 & 450 & 300 \\ 750 & 5000 & 1125 & 750 \\ 450 & 1125 & 1800 & 450 \\ 300 & 750 & 450 & 300 \end{bmatrix}$$

Hence,

Ans : Pseudo-inertia matrix

$$J_i = 100\begin{bmatrix} 8 & 7.5 & 4.5 & 3 \\ 7.5 & 50 & 11.25 & 7.5 \\ 4.5 & 11.25 & 18 & 4.5 \\ 3 & 7.5 & 4.5 & 3 \end{bmatrix}$$

Example 4.2. *Using the L–E formulation determine the equation of motion for the RP manipulator shown in Fig. 4.3.*

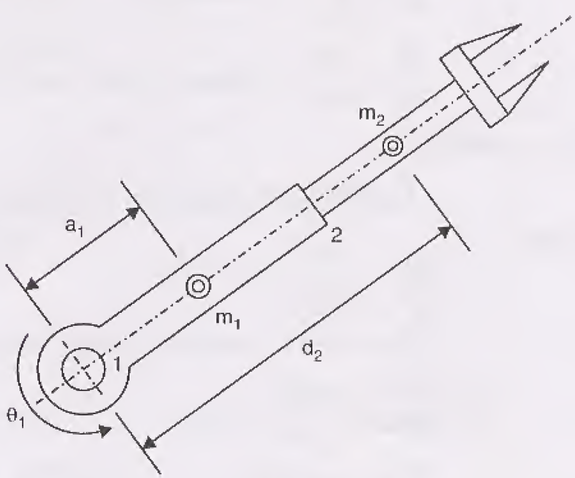


Fig. 4.3. RP Manipulator.

Sol.

- The Kinetic Energies :

The K.E. of link 1, $K_1 = \frac{1}{2} m_1 a_1^2 (\dot{\theta}_1)^2$

The K.E. of link 2, $K_2 = \frac{1}{2} m_2 (d_2^2 \dot{\theta}_1^2 + \dot{d}_2^2)$

Total kinetic energy $K = \frac{1}{2} m_1 a_1^2 (\dot{\theta}_1)^2 + \frac{1}{2} m_2 (d_2^2 \dot{\theta}_1^2 + \dot{d}_2^2)$

Total Potential Energies.

The P.E. of link 1, $P_1 = m_1 a_1 g \sin(\theta_1)$

The P.E. of link 2, $P_2 = m_2 g d_2 \sin(\theta_1)$

Total potential energy

$$P = + m_1 a_1 g \sin(\theta_1) + m_2 g d_2 \sin(\theta_1)$$
$$= g(m_1 a_1 + m_2 d_2) \sin(\theta_1)$$

- The Lagrangian-Euler formulation

Using the relation (4.20)

$$Q_L = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q}$$

For $q = \theta_1$

$$\frac{\partial K}{\partial \dot{q}} = \frac{\partial K}{\partial \dot{\theta}_1} = m_1 a_1^2 \dot{\theta}_1 + m_2 d_2^2 \dot{\theta}_1$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_1}\right) = (m_1 a_1^2 + m_2 d_2^2)\ddot{\theta}_1 \quad \text{...(a)}$$

$$\frac{\partial K}{\partial q} = \frac{\partial K}{\partial \theta_1} = -2 m_2 d_2 \dot{\theta}_1 \dot{d}_2 \quad \text{...(b)}$$

$$\frac{\partial P}{\partial q} = \frac{\partial P}{\partial \theta_1} = + g(m_1 a_1 + m_2 d_2) \cos(\theta_1) \quad \text{...(c)}$$

From relations (a), (b) and (c)

$$T_1 = (m_1 a_1^2 + m_2 d_2^2)\ddot{\theta}_1 + g(m_1 a_1 + m_2 d_2) \cos \theta_1$$

Now assuming $q = d_2$

$$\frac{\partial K}{\partial \dot{d}_2} = m_2 \dot{d}_2$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{d}_2}\right) = m_2 \ddot{d}_2$$

$$\frac{dK}{\partial d_2} = + m_2 d_2 \dot{\theta}_1^2$$

$$\frac{\partial P}{\partial d_2} = m_2 g \sin(\theta_1).$$

Substituting in equation

$$T_2 = F_2 = m_2 \ddot{d}_2 + m_2 g \sin(\theta_1).$$

Hence inertia terms $\quad D_{ij} = \begin{bmatrix} m_1 a_1^2 + m_2 d_2^2 & 0 \\ 0 & m_2 \end{bmatrix}$

Centripetal co-efficient and coriolis component

$$D_{ijk} = \begin{bmatrix} 2m_2 d_2 \dot{\theta}_1 \dot{d}_2 \\ -m_2 d_2 \dot{\theta}_1^{\,2} \end{bmatrix}$$

Gravity terms $\quad D_i = \begin{bmatrix} (m_1 a_1 + m_2\, a_1)\, g \cos(\theta_1) \\ d_2 m_2 g \sin(\theta_1) \end{bmatrix}.$

Example 4.3. *Compute the effective inertia, coupling inertia, centripetal and coriolis component and the gravity terms for a single link manipular of mass m_1 and length 'a_1' as shown in Fig. 4.4.*

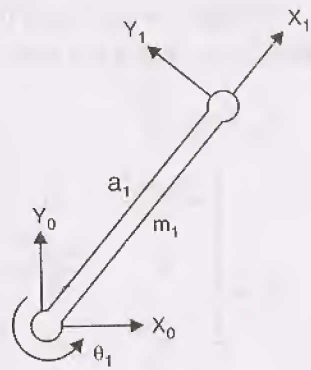


Fig. 4.4. Single Link Manipulator.

Sol.

Kinematic parameters

$$d = 0, \theta = \theta_1$$
$$a = a_1, \alpha = 0$$

Joint variables, $\quad q_i = \theta_1$

- Computation of inertia terms from equation (4.49)

$$D_{ij} = \sum_{i,j}^{6} \text{Trace}\left(\frac{\partial H_P}{\partial q_j} J_P \frac{\partial H_P^{\,T}}{\partial q_i} \right)$$

As $q_i = \theta_1$ the above relation reduces to the relation

$$D_{11} = \text{Trace}\left(\frac{\partial H_1}{\partial \theta_1} . J_1 . \frac{\partial H_1^{\,T}}{\partial \theta_1} \right)$$

where $H_1 = \begin{bmatrix} c1 & -s1 & 0 & a_1 c1 \\ s1 & c1 & 0 & a_1 s1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Differentiating partially w.r.t. θ_1

$$\frac{\partial H_1}{\partial \theta_1} = \begin{bmatrix} -s1 & -c1 & 0 & -a_1 s1 \\ c1 & -s1 & 0 & a_1 c1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\frac{\partial H_1^T}{\partial \theta_1} = \begin{bmatrix} -s1 & c1 & 0 & 0 \\ -c1 & -s1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_1 s1 & a_1 c1 & 0 & 0 \end{bmatrix}$$

Using the equation for J_i in (4.42)

$$J_1 = \begin{bmatrix} -I_{xx} & 0 & 0 & 0 \\ 0 & I_{xx} & 0 & 0 \\ 0 & 0 & I_{xx} & 0 \\ 0 & 0 & 0 & m_1 \end{bmatrix}$$

In the relation for J_i $I_{yy} = I_{zz} = 0$ and $I_{xy} = I_{yz} = I_{zx} = 0$, also $m_1\bar{x}_i = m_1\bar{y}_i = m_1\bar{z}_i = 0$. Since it has to be assumed that $l = a_1$ and $b = h = 0$,

$$I_{xx} = m_1 \frac{a_1^2}{3}$$

$$J_1 = \begin{bmatrix} \frac{-m_1 a_1^2}{3} & 0 & 0 & 0 \\ 0 & \frac{m_1 a_1^2}{3} & 0 & 0 \\ 0 & 0 & \frac{m_1 a_1^2}{3} & 0 \\ 0 & 0 & 0 & m_1 \end{bmatrix}$$

$$\frac{\partial H_1}{\partial \theta_1} . J_1 . \frac{\partial H_1^T}{\partial \theta_1} = \begin{bmatrix} -s1 & -c1 & 0 & -a_1 s1 \\ c1 & -s1 & 0 & a_1 c1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{m_1 a_1^2}{3} & & & \text{zero} \\ & \frac{m_1 a_1^2}{3} & & \\ & & \frac{m_1 a_1^2}{3} & \\ \text{zero} & & & m_1 \end{bmatrix}$$

$$\begin{bmatrix} -s1 & c1 & 0 & 0 \\ -c1 & -s1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_1 s1 & a_1 c1 & 0 & 0 \end{bmatrix}$$

$$\text{Trace}\left[\frac{\partial H_1}{\partial \theta_1} . J_1 . \frac{\partial H_1{}^T}{\partial \theta_1}\right] = \frac{-(s1)^2\, m_1 a_1{}^2}{3} + \frac{(c1)^2\, m_1 a_1{}^2}{3} - \frac{a_1{}^2\, m_1 a_1{}^2\,(s1)^2}{3}$$

$$+ \frac{(s1)^2\, m_1 a_1{}^2}{3} - \frac{a_1{}^2 m_1 (c1)^2}{3} + \frac{a_1{}^2\, m_1 a_1{}^2\,(s1)^2}{3} + \frac{m_1 a_1{}^2}{3}$$

$$= . \frac{m_1 a_1{}^2}{3}$$

The computation of centripetal and Coriolis terms

Using the relation (4.50)

$$D_{ijk} = \sum_{ijk}^{6} \text{Trace}\left(\frac{\partial^2 H_P}{\partial \theta_1 . \partial \theta_1} . J_P . \frac{\partial H_P{}^T}{\partial \theta_1}\right)$$

$$\frac{\partial^2}{\partial \theta_1}\left(\frac{\partial H_1}{\partial \theta_1}\right) = \begin{bmatrix} -c1 & s1 & 0 & -a_1 c1 \\ -s1 & -c1 & 0 & -a_1 s1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and $$\sum_{1}^{6} \text{Trace}\left(\frac{\partial^2 H_1}{\partial \theta_1{}^2} . J_1 . \frac{\partial H_1{}^T}{\partial \theta_1}\right) = 0.$$

The computation of gravity terms

$$D_i = \sum_{1}^{6} - m_i g^T \frac{\partial H_P}{\partial q_i} \bar{r}_P$$

$$m_i = m_1, \quad g^T = [0, +g, +0, 0]$$

$$\bar{r}_P = \left[0, \frac{a_1}{2}, 0, 0\right]^T, \frac{\partial H_P}{\partial q_i} = \frac{\partial H_1}{\partial \theta_1}$$

$$D_1 = -m_1 \begin{bmatrix} 0 \\ +g \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -s1 & -c1 & 0 & -a_1 s1 \\ c1 & -s1 & 0 & a_1 c1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{a_1}{2} \\ 0 \\ 0 \end{bmatrix}$$

Hence, $$D_1 = \frac{m_1 g a_1 c1}{2} = \text{gravity term.}$$

Note. The mass is assumed to act at CG of link.

Example 4.4. *Derive a dynamic model for a two axis planar articulated robot given the mass and length of the links.*

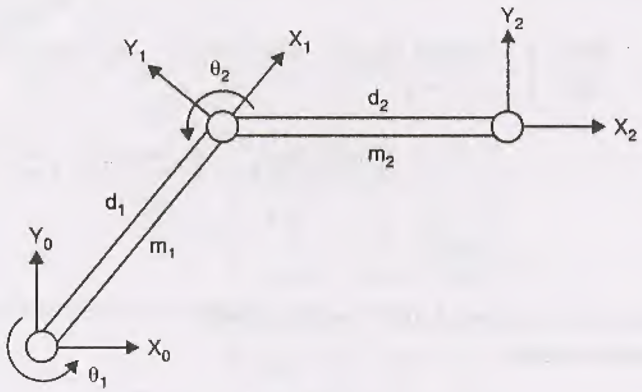


Fig. 4.5

Sol.

The dynamic equation for a general 'n' link manipulator is given by

$$F_i = \sum_{1}^{n} D_{ij} \cdot \ddot{q}_j + I_{ai}\, \ddot{q}_i + \sum_{1}^{n} \sum_{1}^{n} D_{ijk}\, \dot{q}_j \dot{q}_k + D_i$$

$$\sum_{1}^{n} D_{ij} = \sum_{j=i}^{n} \sum_{k=1}^{j} \text{Trace}\left(\frac{\partial H_j}{\partial q_k} J_j \frac{\partial H_j^{T}}{\partial q_i} \right)$$

Case 1 : $i = 1$ and $j = 1$ and $k = 1$

$$\sum_{1}^{1} D_{11} = \sum_{1}^{1} \sum_{k=1}^{1} \text{Trace}\left(\frac{\partial H_1}{\partial \theta_1} J_1 \frac{\partial H_1^{T}}{\partial \theta_1} \right)$$

$$H_1 = \begin{bmatrix} c_1 & -s_1 & 0 & d_1c_1 \\ s_1 & c_1 & 0 & d_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \frac{\partial H_1}{\partial \theta_1} = \begin{bmatrix} -s_1 & -c_1 & 0 & -d_1s_1 \\ c_1 & -s_1 & 0 & d_1c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The inertia matrix is given by

$$J_1 = \begin{bmatrix} \frac{-m_1d_1^{2}}{3} & & & \\ & \frac{m_1d_1^{2}}{3} & & \\ & & \frac{m_1d_1^{2}}{3} & \\ & & & m_1 \end{bmatrix}$$

$$\frac{\partial H_1^{T}}{\partial \theta_1} = \begin{bmatrix} -s_1 & c_1 & 0 & 0 \\ -c_1 & -s_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d_1s_1 & d_1c_1 & 0 & 0 \end{bmatrix}$$

But Trace $A_{ij} = \Sigma A_{ii}$

$$\text{Trace}\left(\frac{\partial H_1}{\partial\theta_1} J_1 \frac{\partial H_1^T}{\partial\theta_1}\right)$$

$$= \frac{-m_1 d_1}{3} s_1^2 + \frac{m_1 d_1}{3} c_1^2 + m_1 d_1^2 s_1^2 - \frac{m_1 d_1^2}{3} c_1^2$$

$$= + \frac{m_1 d_1^2}{3} s_1^2 + m_1 d_1^2 c_1^2$$

$$= m_1 d_1^2 (s_1^2 + c_1^2) = m_1 d_1^2$$

Case 2 : $i = 1, j = 2$ **and** $k = 1$.

$$\Sigma D_{12} = \text{Trace}\left(\frac{\partial H_2}{\partial\theta_1} J_2 \frac{\partial H_2^T}{\partial\theta_1}\right)$$

But
$$H_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & d_1c_1 + d_2c_{12} \\ s_{12} & c_{12} & 0 & d_1s_1 + d_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial H_2}{\partial\theta_1} = \begin{bmatrix} -s_{12} & -c_{12} & 0 & -d_1s_1 - d_2s_{12} \\ c_{12} & -s_{12} & 0 & d_1c_1 + d_2c_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \frac{-m_2 d_2^2}{3} & & & \\ & \frac{m_2 d_2^2}{3} & & \\ & & \frac{m_2 d_2^2}{3} & \\ & & & m_2 \end{bmatrix}$$

$$\frac{\partial H_2^T}{\partial\theta_1} = \begin{bmatrix} -s_{12} & c_{12} & 0 & 0 \\ -c_{12} & -s_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d_1s_1 - d_2s_{12} & d_1c_1 + d_2c_{12} & 0 & 0 \end{bmatrix}$$

$$\text{Trace}\left(\frac{\partial H_2}{\partial\theta_1} . J_2 . \frac{\partial H_2^T}{\partial\theta_1}\right)$$

$$= -\frac{m_2 d_2^2}{3} (s_{12})^2 + \frac{m_2 d_2^2}{3} (c_{12})^2 + (d_1s_1 + d_2s_{12})^2 m_2$$

$$- \frac{m_2 d_2^2}{3} (c_{12})^2 + \frac{m_2 d_2^2}{3} (s_{12})^2 + (d_1c_1 + d_2c_{12})^2 m_2$$

$$= 2d_1d_2s_1s_{12}m_2 + 2d_1d_2c_1c_{12}m_2.$$

Case 3 : $i = 2, j = 2, k = 2$

$$\Sigma D_{22} = \text{Trace}\left(\frac{\partial H_2}{\partial \theta_2} J_2 \frac{\partial H_2^T}{\partial \theta_2}\right)$$

$$\frac{\partial H_2}{\partial \theta_2} = \begin{bmatrix} -s_{12} & -c_{12} & 0 & -d_2 s_{12} \\ c_{12} & -s_{12} & 0 & d_2 c_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial H_2^T}{\partial \theta_2} = \begin{bmatrix} -s_{12} & c_{12} & 0 & 0 \\ -c_{12} & -s_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d_2 s_{12} & d_2 c_{12} & 0 & 0 \end{bmatrix}$$

$$\text{Trace}\left(\frac{\partial H_2}{\partial \theta_2} . J_2 . \frac{\partial H_2^T}{\partial \theta_2}\right) = \frac{-m_2 d_2^2}{3}(s_{12})^2 + \frac{m_2 d_2}{3}(c_{12})^2$$

$$+ (-d_2 s_{12})^2 m_2 - \frac{m_2 d_2^2}{3}(c_{12})^2 + \frac{m_2 d_2^2}{3}(s_{12})^2 + (d_2 c_{12})^2 m_2$$

$$\Sigma D_{22} = m_2 d_2^2 (s_{12}^2 + c_{12}^2) = m_2 d_2^2.$$

- **To Compute D_{ijk}**

$$\sum_1^n \sum_1^n D_{ijk} = \sum_{i,j,k}^n \text{Trace}\left(\frac{\partial^2 H_j}{\partial q_j \partial q_k} J_j \frac{\partial H_j^T}{\partial q_i}\right)$$

Case 1 : $i = 1, j = 1, k = 1$

$$D_{111} = \text{Trace}\left(\frac{\partial^2 H_j}{\partial q_1 \partial q_1} J_j \frac{\partial H_j^T}{\partial q_1}\right).$$

$$\text{Trace}\left(\frac{\partial^2 H_1}{\partial \theta_1^2} . J_1 \frac{\partial H_1^T}{\partial \theta_1}\right)$$

$$\frac{\partial^2 H_1}{\partial \theta_1^2} = \begin{bmatrix} -c_1 & s_1 & 0 & -d_1 c_1 \\ -s_1 & -c_1 & 0 & -d_1 s_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{111} = -\frac{m_1 d_1^2}{3} c_1 s_1 - \frac{m_1 d_1^2}{3}(c_1 s_1) + d_1^2 c_1 s_1 m_1$$

$$+ \frac{m_1 d_1^2}{3}(c_1 s_1) + \frac{m_1 d_1^2}{3}(c_1 s_1) - d_1^2 c_1 s_1 m_1$$

$$= 0.$$

Case 2 : $i = 1, j = 2, k = 2$

$$D_{122} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial^2 \theta_2} J_2 \frac{\partial H_2^T}{\partial \theta_1}\right)$$

$$\frac{\partial^2 H_2}{\partial \theta_2{}^2} = \begin{bmatrix} -c_{12} & s_{12} & 0 & -d_2c_{12} \\ -s_{12} & -c_{12} & 0 & -d_2s_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{122} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial \theta_2{}^2} \cdot J_2 \frac{\partial H_2{}^T}{\partial \theta_1}\right)$$

$$= -\frac{m_2d_2{}^2}{3} c_{12}s_{12} - \frac{m_2d_2{}^2}{3} c_{12}s_{12} + m_2d_2c_{12}(d_1s_1 + d_2s_{12}) + \frac{m_2d_2{}^2}{3} c_{12}s_{12} + \frac{m_2d_2{}^2}{3} c_{12}s_{12} - m_2d_2s_{12}(d_1c_1 + d_2c_{12})$$

$$= m_2d_2\,[d_1s_1c_{12} - d_1c_1s_{12}]$$

Case 3 : To compute D_{121}
Assume $i = 1, j = 2$, and $k = 1$

$$D_{121} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial \theta_1 \partial \theta_2} \cdot J_2 \cdot \frac{\partial H_2{}^T}{\partial \theta_1}\right).$$

$$\frac{\partial^2 H_2}{\partial \theta_2 \partial \theta_1} = \begin{bmatrix} -c_{12} & s_{12} & 0 & -d_2c_{12} \\ -s_{12} & -c_{12} & 0 & -d_2s_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{121} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial \theta_2 \partial \theta_1} J_2 \frac{\partial H_2{}^T}{\partial \theta_1}\right)$$

$$= -\frac{m_2d_2{}^2}{3} c_{12}\,s_{12} - \frac{m_2d_2{}^2}{3} c_{12}s_{12} + m_2d_2c_{12}\,(d_1s_1 + d_2s_{12}) + \frac{m_2d_2{}^2}{3} c_{12}s_{12} + \frac{m_2d_2{}^2}{3} c_{12}s_{12} - m_2d_2s_{12}\,(d_1c_1 + d_2c_{12})$$

$$= m_2d_2\,(d_1s_1c_{12} - d_1c_1s_{12})$$

Case 4 : To compute D_{222}
Assume $i = 2, j = 2$ and $k = 2$

$$D_{222} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial \theta_2{}^2} \cdot J_2 \cdot \frac{\partial H_2{}^T}{\partial \theta_2}\right)$$

$$\frac{\partial^2 H_2}{\partial \theta_2{}^2} = \begin{bmatrix} -c_{12} & s_{12} & 0 & -d_2c_{12} \\ -s_{12} & -c_{12} & 0 & -d_2s_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial H_2^T}{\partial \theta_2} = \begin{bmatrix} -s_{12} & c_{12} & 0 & 0 \\ -c_{12} & -s_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d_2 s_{12} & d_2 c_{12} & 0 & 0 \end{bmatrix}$$

$$D_{222} = -\frac{m_2 d_2^2}{3} c_{12} s_{12} - \frac{m_2 d_2^2}{3} c_{12} s_{12} + m_2 d_2^2 c_{12} s_{12}$$

$$+ \frac{m_2 d_2^2}{3} c_{12} s_{12} + \frac{m_2 d_2^2}{3} c_{12} s_{12} - m_2 d_2^2 c_{12} s_{12}$$

$$= 0.$$

Case 5 : To compute D_{221}, assume $i = 2, j = 2, k = 1$

$$D_{221} = \text{Trace}\left(\frac{\partial^2 H_2}{\partial \theta_2\, \partial \theta_1} J_2 \frac{\partial H_2^T}{\partial \theta_2}\right)$$

$$-\frac{m_2 d_2^2}{3} c_{12} s_{12} - \frac{m_2 d_2^2}{3} c_{12} s_{12} + m_2 d_2^2 c_{12} s_{12}$$

$$+ \frac{m_2 d_2^2}{3} c_{12} s_{12} + \frac{m_2 d_2^2}{3} c_{12} s_{12} - m_2 d_2^2 c_{12} s_{12}$$

$$= 0.$$

- **Computation of Gravity Terms**

$$\Sigma D_i = \Sigma - m_i\, g_i^T \frac{\partial H_i}{\partial q_i} \bar{r}_i$$

Case 1: $i = 1, q_i = \theta_1$

Hence

$$D_1 = -m_1 \begin{bmatrix} 0 \\ +g \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -s_1 & -c_1 & 0 & -d_1 s_1 \\ c_1 & -s_1 & 0 & d_1 c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ d_1 \\ 0 \\ 0 \end{bmatrix}$$

$$D_1 = m_1 g d_1 \sin(\theta_1).$$

Case 2: $i = 1,\ q_i = \theta_2$.

$$D_2 = -m_2 \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -s_{12} & -c_{12} & 0 & -d_2 s_{12} \\ c_{12} & -s_{12} & 0 & d_2 c_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ d_1 + d_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= m_2 (d_1 + d_2) g. \sin(\theta_1 + \theta_2).$$

Example 4.5. *Derive, using matrix method of L-E formulation, the expressions for the inertia term co-efficients and the gravity terms, for the RP manipulator shown in Fig. 4.3.*

Sol.

(*a*) The Co-efficients for the Inertia Terms

$$D_{ij} = \text{Trace}\left(\frac{\partial H_j}{\partial q_j} J_j \frac{\partial H_j^T}{\partial q_i}\right).$$

Case 1 : Computation of D_{11} assuming $i = 1, j = 1$ and $q_i = \theta_1$

So

$$D_{11} = \text{Trace}\left(\frac{\partial H_1}{\partial \theta_1} J_1 \frac{\partial H_1^T}{\partial \theta_1}\right)$$

$$H_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial H_1}{\partial \theta_1}\begin{bmatrix} -s_1 & -c_1 & 0 & -a_1s_1 \\ c_1 & -s_1 & 0 & a_1c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \frac{\partial H_1^T}{\partial \theta_1}\begin{bmatrix} -s_1 & c_1 & 0 & 0 \\ -c_1 & -s_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -a_1s_1 & a_1c_1 & 0 & 0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \frac{-m_1a_1^2}{3} & & \text{zero} & \\ & \frac{m_1a_1^2}{3} & & \\ \text{zero} & & \frac{m_1a_1^2}{3} & \\ & & & m_1 \end{bmatrix}$$

$$\text{Trace}\left(\frac{\partial H_1}{\partial \theta_1} . J_1 \frac{\partial H_1^T}{\partial \theta_1}\right) = -\frac{m_1a_1^2}{3}(s_1)^2 + \frac{m_1a_1^2}{3}(c_1)^2 + m_1a_1^2s_1^2$$

$$+ \frac{m_1a_1^2}{3}(s_1)^2 - \frac{m_1a_1^2}{3}(c_1)^2 + m_1a_1^2c_1^2$$

$$= m_1a_1^2(s_1^2 + c_1^2)$$

$$= m_1a_1^2 \text{ ---------- as } (s_1^2 + c_1^2) = 1.$$

Hence $\quad D_{11} = m_1a_1^2.$

Case 2 : Computation of D_{12} assuming $i = 1,\ j = 2$ and $q_1 = \theta_1$.

$$D_{12} = \text{Trace}\left(\frac{\partial H_2}{\partial d_2} . J_2 . \frac{\partial H_2^T}{\partial \theta_1}\right)$$

$$H_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{\partial H_2}{\partial d_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial H_2}{\partial \theta_1} \begin{bmatrix} -s_1 & -c_1 & 0 & -a_1s_1 \\ c_1 & -s_1 & 0 & a_1c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \frac{\partial H_2{}^T}{\partial \theta_1} \begin{bmatrix} -s_1 & c_1 & 0 & 0 \\ -c_1 & -s_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -a_1s_1 & a_1c_1 & 0 & 0 \end{bmatrix}$$

$$D_{12} = \text{Trace}\left(\frac{\partial H_2}{\partial d_2} . J_2 . \frac{\partial H_2{}^T}{\partial \theta_1}\right).$$

$$\text{where } J_2 = \begin{bmatrix} \frac{-m_2d_2{}^2}{3} & & & \\ & \frac{m_2d_2{}^2}{3} & & \\ & & \frac{m_2d_2{}^2}{3} & \\ & & & m_2 \end{bmatrix}$$

$D_{12} = 0.$

Case 3 : Computation of D_{22}

Assuming $i = 2,\ j = 2$ and $q_2 = d_2$

$$D_{22} = \text{Trace}\left(\frac{\partial H_2}{\partial d_2} . J_2 . \frac{\partial H_2{}^T}{\partial d_2}\right)$$

$$\frac{\partial H_2{}^T}{\partial d_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Hence, $D_{22} = m_2$

(*b*) **Gravity Terms**

$$D_i = -m_i.g_i{}^T. \frac{\partial H_i}{\partial q_i}.\bar{r}_i$$

when $i = 1$

$$D_1 = -m_1g_1{}^T \frac{\partial H_1}{\partial \theta_1}.\bar{r}_1$$

$$D_1 = -m_1 \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -s_1 & -c_1 & 0 & -a_1s_1 \\ c_1 & -s_1 & 0 & a_1c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a_1 \\ 0 \\ 0 \end{bmatrix}$$

$$= m_1a_1g \cos \theta_1$$

when $i = 2$
$$D_2 = -m_2 g^T \frac{\partial H_2}{\partial \theta_1} ; \bar{r}_2$$

$$D_2 = -m_2 \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -s_1 & -c_1 & 0 & -a_1 s_1 \\ c_1 & -s_1 & 0 & a_1 c_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_2 \\ a_1 \\ 0 \\ 0 \end{bmatrix}$$

$$= m_2 d_2 g \sin \theta_1 + m_2 a_1 g \cos \theta_1.$$

Example 4.6. *The force and the torque in the base co-ordinate frame is given by*

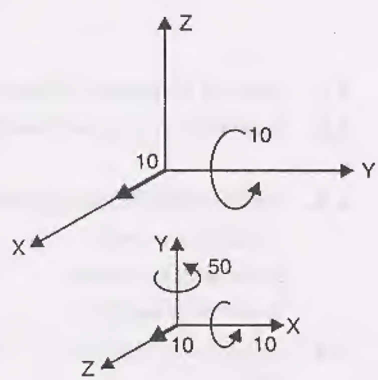
$$f = 10i + 0j + 0k$$
$$m = 0i + 10j + 0k$$

Determine the force and the torque at the tool tip frame described by

$$A = \begin{bmatrix} 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Sol.
$$n = 0i + 1j + 0k$$
$$o = 0i + 0j + 1k$$
$$a = 1i + 0j + 0k$$
$$p = 10i + 5j + 0k$$

$$f \times p = \begin{vmatrix} i & j & k \\ 10 & 0 & 0 \\ 10 & 5 & 0 \end{vmatrix} = 0i + 0j + 50k$$



and
$$(f \times p) + m = 0i + 10j + 50k$$
$$m_x = n \,.\, ((f \times p) + m)$$
$$= 10$$
$$m_y = 50$$
$$m_z = 0$$
$$m = 10i + 50j + 0k$$
$$f_x = n \,.\, f = 0$$
$$f_y = 0 \,.\, f = 0$$
$$f_z = a \,.\, f = 10$$
$$f = 0i + 0j + 10k$$

Example 4.7. *The end effector of a manipulator is to exert a force of $f = 0i + 0j + 10k$ and a moment of $m = 0i + 0j - 100k$. The position of the end effector is described by*

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine the equivalent moment and force at the wrist ?

Sol.

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n = 1i + 0j + 0k$$

$$o = 0i + 1j + 0k$$

$$a = 0i + 0j + 1k$$

$$p = -2i + 0j - 10k.$$

$$f \times p = \begin{vmatrix} i & j & k \\ 0 & 0 & 10 \\ -2 & 0 & -10 \end{vmatrix} = 0i + (-20)j + 0k$$

$$(f \times p) + m = 0i - 20j - 100k$$

$$f_x = n \,.\, f = 0 \quad f_y = 0 \,.\, f = 0 \quad f_2 = a \,.\, f = 10$$

Hence $$f = [0i + 0j + 10k]$$

$$m_x = n \,.\, ((f \times p) + m) = 0$$

$$m_y = 0 \,.\, ((f \times p) + m) = -20$$

$$m_z = a \,.\, ((f \times p) + m) = -100$$

$$w_m = [0i - 20j - 100k].$$

EXERCISE

4.1. Discuss Lagrange–Euler formulations for a robotic manipulator. *(VTU–Jan./Feb. 2004)*

4.2. Explain Lagrange–Euler formulation for a robot arm.
(VTU–May/June–2004 ; VTU–Jan./Feb. 2003)

4.3. Explain the following briefly as applied to robot arm dynamics analysis.

1. Kinetic energy.
2. Potential energy.
3. Joint velocities.

4.4. Derive the equation of motion for a single link manipulator given the mass and length of the link.

4.5. What is Lagrangian ? Give the derivation of Lagrange-Euler formulation for the joint force/torque.

4.6. Derive the dynamic equation of motion for a Revolute-Prismatic (RP) robot arm manipulator.

4.7. Explain following as applied to a robot arm, and also discuss their importance.

(*a*) Inertia terms.

(*b*) Coupling inertia.

(*c*) Centrifugal force.

(*d*) Coriolis component.

(*e*) Gravity term.

4.8. It is given that a two link manipulator has to exert a force of $f = 10i + 5j$ at its tool tip. The Jacobian for the manipulator is given by

$$J = \begin{bmatrix} -(l_1 S_1 + l_2 S_{12}) & -l_2 S_{12} \\ (l_1 C_1 + l_2 C_{12}) & l_2 C_{12} \end{bmatrix}$$

Given that $l_1 = 12$ $l_2 = 10$, $\theta_1 = 30°$ and $\theta_2 = 45°$

Determine the joint torques.

5 Robot Control System

5.1 INTRODUCTION TO CONTROL CONCEPTS

The robot joints are driven by the actuators which act on the command signals issued to follow the desired input and to produce the required output. This procedure is known as 'robot control' problem. For the robot to perform the pre-determined criteria, given the dynamic equations of motion, the drive mechanisms have to be controlled using the systems called controllers which achieve dynamic responses of the manipulators. There are mainly two classified approaches to the robot control problem.

- **Approach 1**

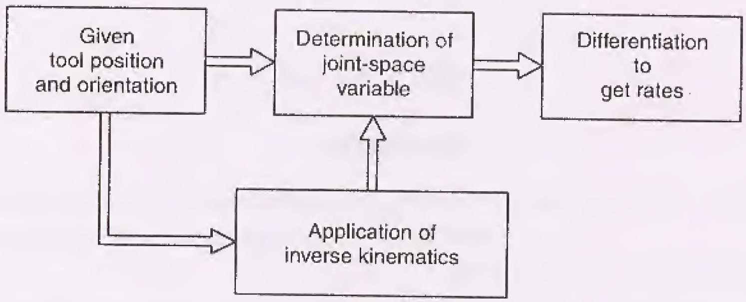


Fig. 5.1

The tool position and orientation, $[n, s, a, p]$ at a given time, is determined by the configuration of the object with respect to the base reference frame. By application of inverse kinematics solution it is possible to determine the joint-space variables $[\theta_1, \theta_2, ...]$. The differentiation of joint-space variables yields the rates—joint velocity and accelerations, at which the actuators should operate to control the robot motion receiving the signal from the control system.

In this approach to the control problem, there is a need to develop a dynamic model using inertia co-efficients, Centrifugal and Coriolis force and the gravity force to determine the joint torque/force.

Further the joint forces are equated to the differential equation representing the control law with mass, friction damping and the stiffness partitions. Based on the control law, the system response and the performances are determined. This solution throws light on the vibrational and the stability aspect of the manipulator.

• Approach 2

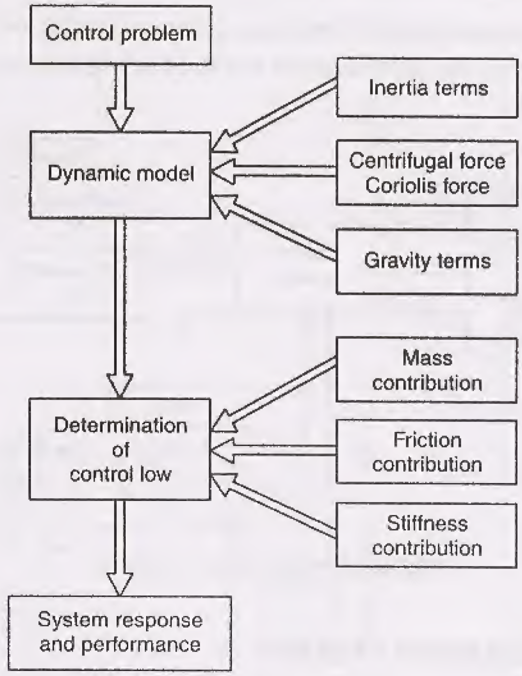


Fig. 5.2

The motion of the robot arm analysed from the control point of view has two separate phases.

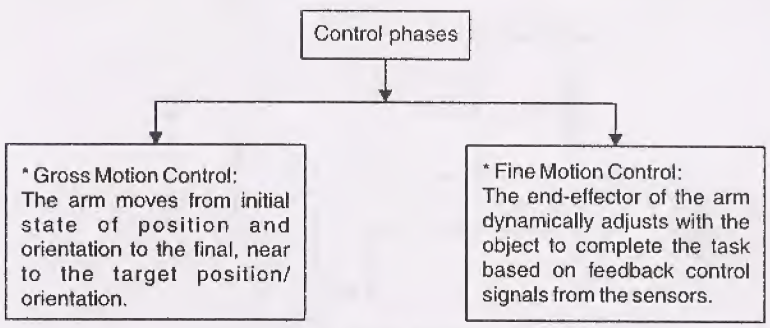


Fig. 5.3

The exsisting industrial practice is to design the control system considering the simple joint servo mechanism to each joint in the manipulator rather than applying the control strategy to the whole robot arm which complicats the procedure. This results in limited speed and accuracy of the end-effector. The concept limits the robot application to the fewer tasks.

5.2 BLOCK DIAGRAM OF ROBOT CONTROL SYSTEM

A general block representation of the robot control system is as shown in Fig. 5.4. The block diagram establishes the relation between path generator variables with the physical robot functionaries. From the diagram it is clear that the robot motion components can

understand the signals for the joint torque vector from the control system. The controllers read the vector of joint position (θ_1, θ_2 ...), joint velocities ($\dot{\theta}_1, \dot{\theta}_2$, ...) and joint accelerations ($\ddot{\theta}_1, \ddot{\theta}_2$, ...), provided by the manipulator's sensors. If there is difference in the achieved and the desired joint variable vectors, the error signals are fed back to the control system to correct its torque/force output signals.

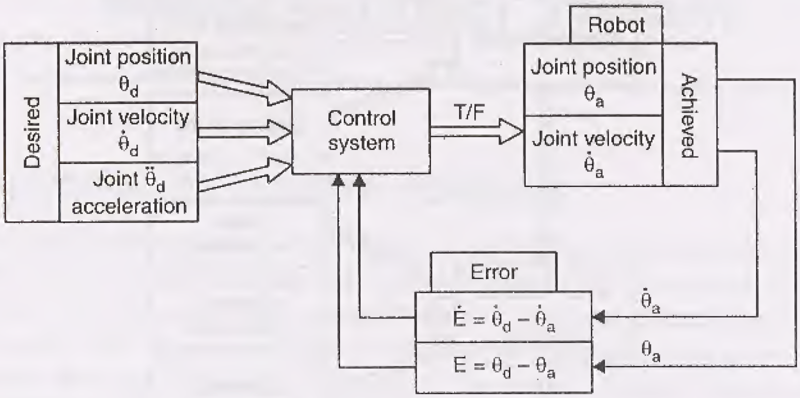


Fig. 5.4. General Block Diagram.

5.3 SPRING-MASS-DAMPER SYSTEM

The analysis of the basic control problem of the manipulator starts in the consideration of simple mechanical system consisting of a spring a mass and a damper. The Fig. 5.5 shows such a system with a block of mass '*m*' attached to a spring of stiffness '*k*' and a damper (friction) with a damping co-efficient '*c*'.

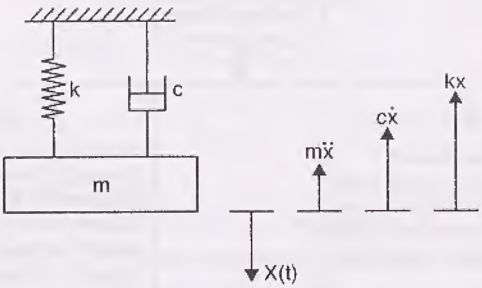


Fig. 5.5. Second Order Linear System.

When the block is disturbed by a distance X(t) the inertial force, the frictional force and the spring force act upward. For free vibration the summation of the three forces equals to zero.

Differential Equation of Motion :

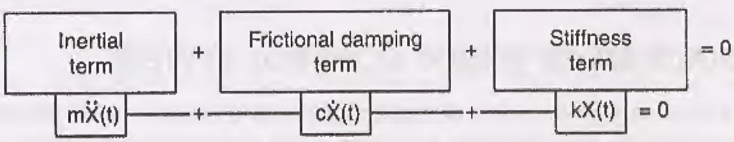
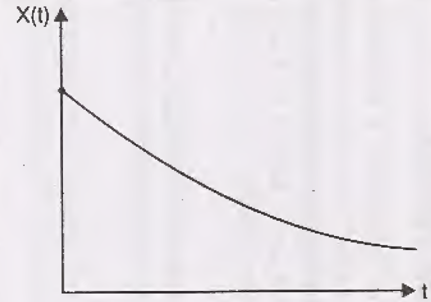
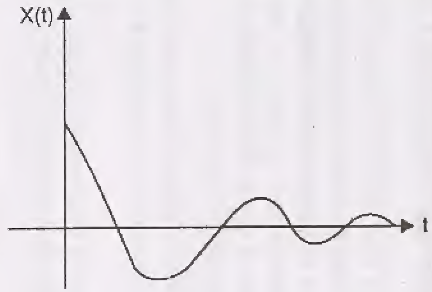
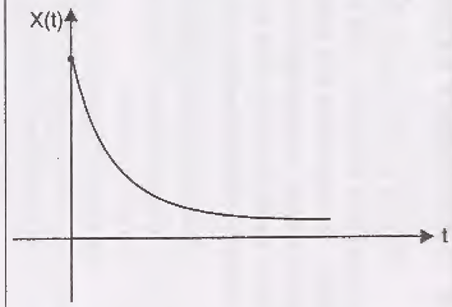


Fig. 5.6

Table 5.1. Responses

Differential Equation of Motion	*Characteristic Equation*	*Roots $S_{1,2}$*	*Type of Roots and Responses*	*Graphical Illustration*
$m\ddot{X} + c\dot{X} + kX = 0$ Differential equation of motion	$mS^2 + cS + k = 0.$ Characteristic equation after Laplace transforms	$S_1 = -\frac{c}{2m} + \frac{\sqrt{c^2 - 4\text{ km}}}{2m}$ $S_2 = -\frac{c}{2m} - \frac{\sqrt{c^2 - 4\text{ km}}}{2m}.$	• Real and unequal • $c^2 > 4$ km • Sluggish behaviour • Friction dominates • Over damped system.	
			• Complex roots, • $c^2 < 4$ km • Oscillatory behaviour • Stiffness dominates • Under damped system.	
			• Real and equal roots • $c^2 = 4$ km • Fast and non-oscillatory response • Friction and stiffness balanced • Critically damped.	

5.4 TRANSIENT RESPONSE OF A SECOND ORDER SYSTEM

When a dynamical system like robotic manipulator, is excited by a suddenly applied non-periodic function, the response to such an excitation is called *'Transient Response'*. In a robotic manipulator the system is of higher order, but for the simplicity of analysis the second order system is assumed for the control analysis of the robot arm. Based on the responses offered by the system the analysis of the control system of a manipulator has two distinctions : (1) Steady-state Analysis (2) Transient Analysis.

The oscillation of a dynamical system die as the time approaches infinity, resulting in a steady state response. This type of behaviour is needed for a system for the stability of operations with change in state.

During the sudden change in states the transition has to take place during which the behaviour of the system is the response known as transient response. The differential equation for a second order system is

$$m\ddot{X} + c\dot{X} + kX = kY \qquad ...(5.1)$$

Taking Laplace Transform

$$mS^2X(S) + cSX(S) + kX(S) = k\dot{Y}(S)$$

or

$$\frac{X(S)}{Y(S)} = \frac{k}{mS^2 + cS + k} \qquad ...(5.2)$$

From the equation (5.2), it is clear that, the natural frequency is

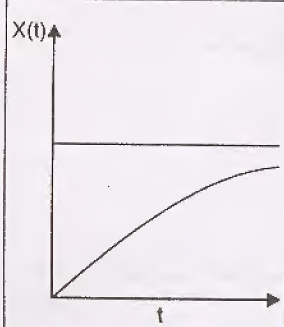
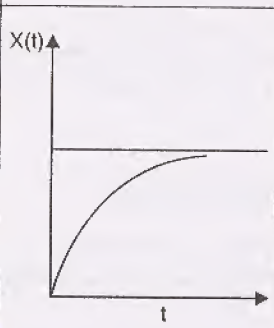
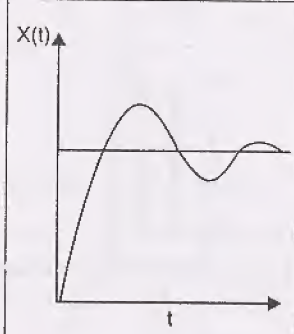
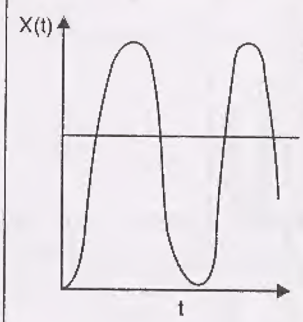
$$w_n = \sqrt{\frac{k}{m}}, \qquad ...(5.3)$$

and the damping ratio is

$$z = \frac{c}{2k} \qquad ...(5.4)$$

For different values of 'z' the transient response is different.

Table 5.2. Transient Responses to Step Input

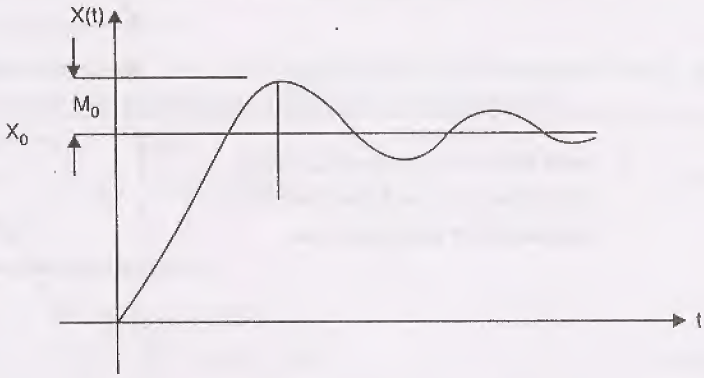
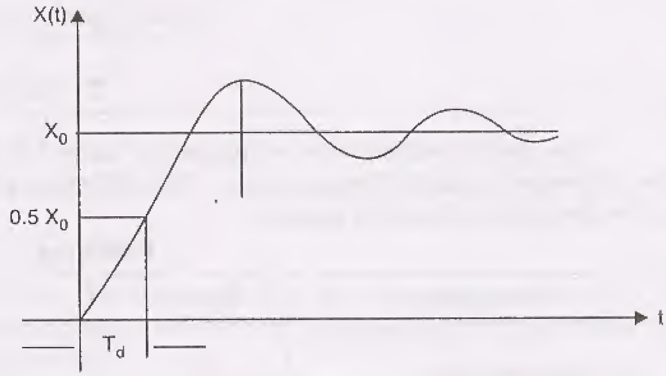
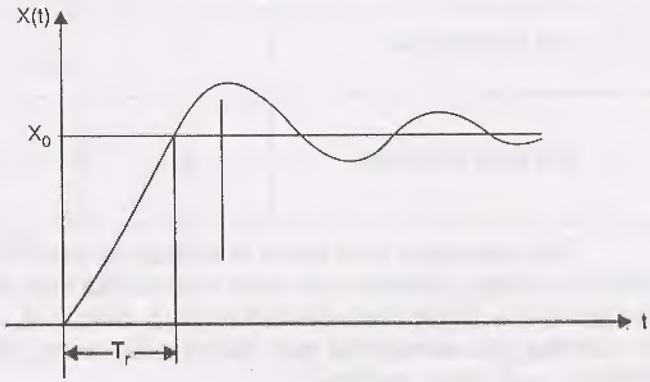
$z > 1$	$z = 1$	$0 < z < 1$	$z = 0$
			
over damped	critically damped	under damped	undamped

- The overdamped system does not vibrate but the response is sluggish.
- The critically damped system reaches steady state very fast and not sluggish, this is desired for a robotic manipulator.

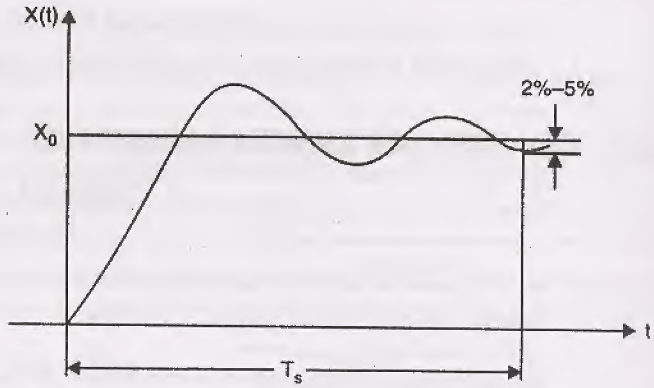
- Under damped system takes longer time to reach steady state.
- Undamped system does not reach steady state at all.

5.5 CONTROLLER DESIGN PARAMETERS

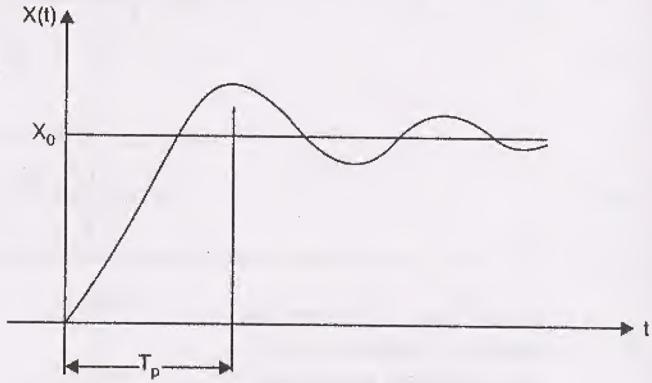
Table 5.3

Parameters	*Illustration*
• Maximum overshoots (M_0) is the maximum peak value measured from the steady state value, X_0.	
• Delay time : (T_d) time that takes the system to reach 50% of steady state value, X_0.	
• Rise time (T_r) time that takes the system to go from 0 to 100% of the final value X_0.	

- Settling time : (T_s) time required for the system to stay within the 2% to 5% of the final value, X_0.



- Peak time : (T_p) time that takes the system to reach the maximum overshoot for the first time.



The design parameters mentioned in table 5.3 depend upon the damping ratio, z and the undamped natural frequency, w_n . The following relation hold good giving fair estimation to the designers of control system.

Table 5.4

Parameters	*Symbol*	*Empiricals*
1. The delay time	T_d	$T_d = \dfrac{1 + 0.6z + 0.15z^2}{w_n}$ for $0 < z < 1$
2. The settling time	T_s	$T_s < \dfrac{4.7}{zw_n}$ for $0 < z < 1$
3. The peak overshoot	M_0	$M_0 = \exp\left(\dfrac{-z\pi}{\sqrt{1-z^2}}\right)$ for $0 < z < 1$.

The maximum over shoot is limited by assuming damping ratio of 0.7 or 0.8 for many control systems. However, in robot control the over shoot has to be completely eliminated by designing the system for critical damping. Here $M_0 = 0$ when the damping ratio is equal to 1. In robotics the overshoots may disturb the work-cell setup and damage the robot itself by collision with other systems.

A reduce the complication of control analysis, a robot with 'n' degrees of freedom is considered as 'n' separate second ordered linear systems.

5.6. CONTROL SYSTEM MODELING

The dynamic model of the robotic manipulator is discussed in detail in Chapter 4. The second ordered system of differential equations are reformulated to first order system of equations called '*state equations*', so as to easen the control of robot arms.

• State-space Model

The state-space Model for the control of the robot manipulator is developed based on both dynamic state equations and kinematic equations for the tool configuration.

Second ordered dynamic equation :

$$\mathrm{D} \cdot \ddot{q}(t) = \mathrm{T} - b\,(q, \dot{q}) - c(\dot{q}) - \mathrm{G}(q) \qquad ...(5.5)$$

First order state-space equation

$$\dot{q}(t) = \mathrm{V} \quad \Rightarrow \quad \text{velocity vector}$$

$$\ddot{q}(t) = \dot{\mathrm{V}} \quad \Rightarrow \quad \text{Acceleration vector}$$

$$= \mathrm{D}^{-1}(q)\,[\mathrm{T} - b(q, \dot{q}) - c(\dot{q}) - \mathrm{G}(q)] \qquad ...(5.6)$$

where

D = positive-definite, non-singular inertia tensor

T = joint torque

b = centrifugal and Coriolis component

c = friction damping function

G = gravity term.

• The Kinematic Output Equation

$$\mathrm{Y} = \mathrm{H}(q). \qquad ...(5.7)$$

Where H(q) is the tool configuration function and Y is the robot output variables of position and orientation. The block diagram representation of the state space model shown in Fig. 5.7 has the following symbolic conversions.

$$\left.\begin{aligned} a &= \dot{\mathrm{V}} = \ddot{q}, \\ \mathrm{V} &= \dot{q}, \\ \text{and} \quad \mathrm{X} &= q(t) \end{aligned}\right] \qquad ...(5.8)$$

But also after transformation

$$\left.\begin{aligned} q(s) &= \mathrm{X}(s) = \frac{\dot{q}(s)}{s} \\ \text{and} \quad \dot{q}(s) &= \mathrm{V}(s) = \frac{\ddot{q}(s)}{s} \\ \text{and} \quad \ddot{q}(s) &= \mathrm{D}^{-1}q(t) = \mathrm{D}^{-1}\mathrm{X}(t). \end{aligned}\right] \qquad ...(5.9)$$

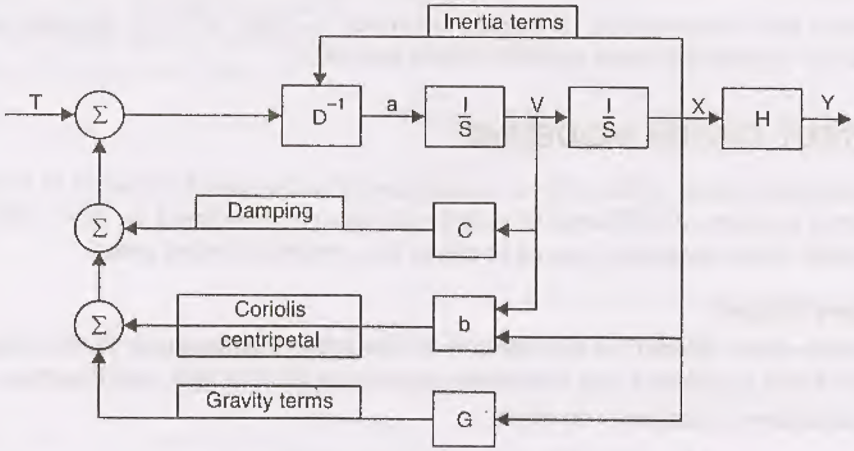
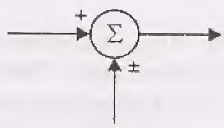
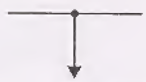


Fig. 5.7. Control System Model.

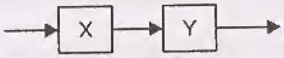
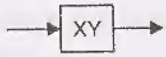
5.7 BLOCK DIAGRAM ELEMENTS

Table 5.5

Sl. No.	*Diagram Elements*	*Illustration*
1.	Transfer function block	$\boxed{\frac{\text{Output}}{\text{Input}} = \frac{Y(s)}{X(s)}}$
2.	Signal arrow	$\xrightarrow{\text{Variable}}$
3.	Summing point	
4.	Take-off point.	

- The function block represents the transfer function which is the ratio of output to input.
- The signal arrow indicates direction of flow of the signal and the corresponding variable of transmission.
- Summing point allows for the algebraic addition of two or more signals.
- Take-off point permits sharing of signals and variables between components.

Table 5.6. Block Reduction Algebra

<table>
<tr><th>Before Reduction</th><th>After Reduction</th></tr>
<tr><td>(1) Blocks in series :

</td><td>
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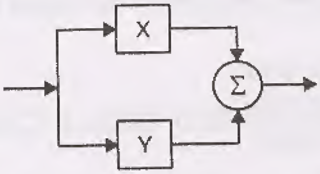
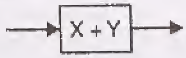
<tr><td>(2) Blocks in parallel :

</td><td>
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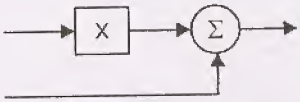
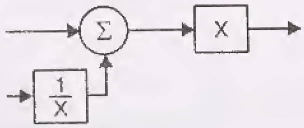
<tr><td>(3) Shifting of summing point :

</td><td>
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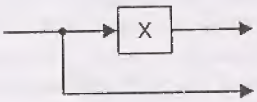
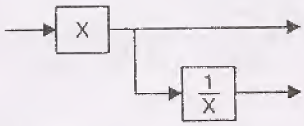
<tr><td>(4) Shifting of take-off point :

</td><td>
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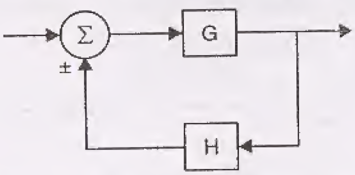
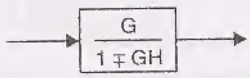
<tr><td>(5) Elimination of feedback loop :

</td><td>
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5.8 CONTROL OF A SINGLE LINK MANIPULATOR

Table 5.7

<table>
<tr><th>Block Reduction and Transfer Function</th><th>Term Description</th></tr>
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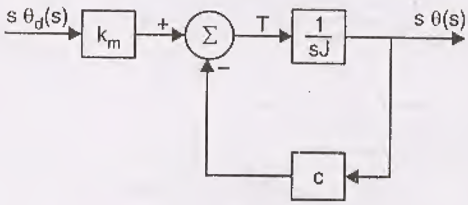
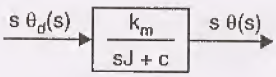
• **Actuator and link transfer function :**





Transfer Function $\dfrac{s\,\theta\,(s)}{s\,\theta_d\,(s)} = \dfrac{k_m}{sJ + c}$

</td><td>

$s\theta_d(s)$–Laplace transform to desired joint velocity $\dot{\theta}_d$

$s\theta(s)$–Laplace transform of the output velocity $\dot{\theta}$

c–damping factor

k_m–actuator gain

T = joint torque.

$J_{ii} = D_{ii} + I_{ai}$ ...(5.10)

= effective link inertia

I_{ai} = actuator inertia

$T = k_m \,.\, \dot{\theta}_d - c \,.\, \dot{\theta}$ ...(5.11)

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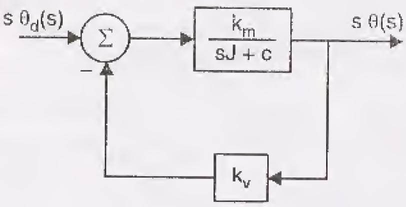
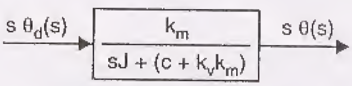
<tr><td>

• **Rate feedback :**





</td><td>

The improvement in natural damping is accomplished by the rate feedback through a tacho-metre generator. k_v is the rate gain. The transfer function of the link and actuator with rate feed back is

$$\frac{s\,\theta\,(s)}{s\,\theta_d\,(s)} = \frac{k_m}{sJ + (c + k_v k_m)} \quad ...(5.12)$$

</td></tr>
</table>

• Position Feedback and Transfer Function

k_p represents the position feedback gain. The encoders provide the feedback information about the position of the link of the manipulator. The block diagram is given by

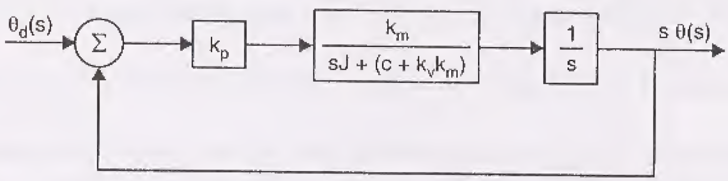


Fig. 5.8

The actuator-link transfer function after rate and position feedback is given by

$$\frac{\theta(s)}{\theta_d(s)} = \frac{k_p k_m}{s^2 J + s(c + k_v\, k_m) + k_p k_m} \quad ...(5.13)$$

The characteristic equation for the second order system is of the form

$$\frac{1}{s^2 + (2zw)s + w^2} \qquad \text{...(5.14)}$$

Comparing this with equation (5.13), we have

$$w = \sqrt{\frac{k_p k_m}{J}}, \qquad \text{....(5.15)}$$

and $$z = \frac{c + k_v k_m}{2(Jk_p k_m)^{1/2}}. \qquad \text{...(5.16)}$$

5.9 MODELING THE TRANSFER FUNCTION FOR A SINGLE JOINT

• Reduction Model

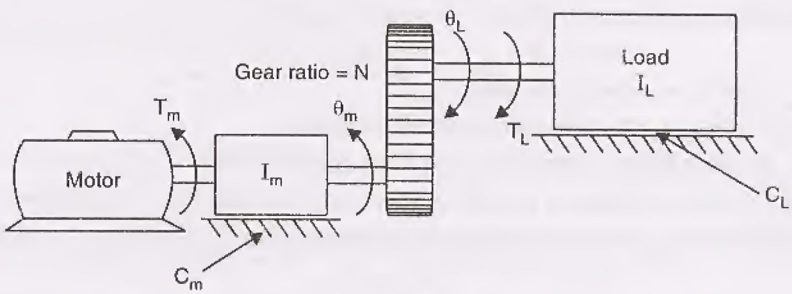


Fig. 5.9. DC Motor Drive.

The Fig. 5.9 shows a load being driven by a DC motor through a gear train with a reduction ratio of N. The following reduction of variables is applicable.

$$\left\{\begin{matrix}\text{Linear distance covered} \\ \text{by the smaller gear}\end{matrix}\right\} = \left\{\begin{matrix}\text{Linear distance covered} \\ \text{by the bigger gear}\end{matrix}\right\}$$

i.e., $$d_m = d_L$$

or $$r_m\, q_m = r_L \,.\, \theta_L$$

where θ_m and θ_L $\Rightarrow$ rotating of motor and Load respectively.

r_m and r_L $\Rightarrow$ radius of gears on motor shaft and load shaft respectively.

But $$n_m \,.\, \theta_m = n_L\, \theta_L$$

$$\frac{\theta_L}{\theta_m} = \frac{n_m}{n_L} = N = \text{gear ratio} \qquad \text{...(5.17)}$$

where n_m and n_L are number of teeth of gears on motor shaft and load shaft respectively.

From equation (5.17) $$\theta_L = N\theta_m \qquad \text{...(5.18)}$$

By time derivatives of θ, $$\dot{\theta}_L = N\dot{\theta}_m \qquad \text{...(5.19)}$$

$$\ddot{\theta}_L = N\ddot{\theta}_m \qquad \text{...(5.20)}$$

- **Torque Model**

$$\begin{Bmatrix}\text{Torque produced}\\ \text{by motor shaft}\end{Bmatrix} = \begin{Bmatrix}\text{Torque dissipated}\\ \text{on motor}\end{Bmatrix} + \begin{Bmatrix}\text{Torque consumed}\\ \text{by load referred}\\ \text{to motor shaft.}\end{Bmatrix}$$

i.e.,
$$T = T_m + (T_L)_m$$
$$= [I_m \ddot{\theta}_m + C_m \dot{\theta}_m] + [I_L\ddot{\theta}_L + C_L\dot{\theta}_L]_m$$

But $\quad (T_L)_m = NT_L$

Hence $\quad T = [I_m \ddot{\theta}_m + C_m \dot{\theta}_m] + N[I_L\ddot{\theta}_L + C_L\dot{\theta}_L]$

Using equations (5.19) and (5.20)

$$T = [I_m \ddot{\theta}_m + C_m \dot{\theta}_m] + N^2 [I_L\ddot{\theta}_m + C_L\dot{\theta}_m]$$
$$= [I_m + N^2 I_L]\, \ddot{\theta}_m + [C_m + N^2 C_L]\, \dot{\theta}_m$$
$$T = I_{eff}\, \ddot{\theta}_m + C_{eff}\, \dot{\theta}_m \qquad ...(5.21)$$

By taking the Laplace transform of equation (5.21)

$$T(s) = s^2\, I_{eff}\, \theta_m(s) + sC_{eff}\, \theta_m(s) \qquad ...(5.22)$$

where $I_{eff} = I_m + N^2 I_L$ = effective inertia

$C_{eff} = C_m + N^2C_L$ = effective damping co-efficient

and I_m and I_L are the inertia at motor and load respectively.

C_m and C_L are the damping co-efficients for motor and load respectively.

But the torque developed at the motor is proportional to armature current i_a and related by

$$T = k_a\, i_a \qquad ...(5.23)$$

Taking Laplace transform, $\quad T(s) = k_a\, i_a(s) \qquad ...(5.24)$

- **Electric Model of Motor**

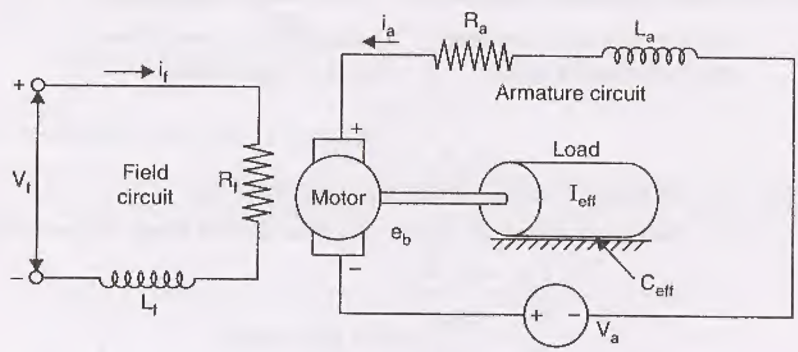


Fig. 5.10. Motor.

A DC motor schematic is as shown in Fig. 5.10.

The symbols employed are

R_a	⇒	resistance	of armature
L_a	⇒	inductance	
i_a	⇒	current	
V_a	⇒	voltage applied	

$$\left.\begin{array}{lll} R_f & \Rightarrow & \text{resistance} \\ L_f & \Rightarrow & \text{Inductance} \\ i_f & \Rightarrow & \text{current} \\ V_f & \Rightarrow & \text{voltage} \end{array}\right] \text{ of field.}$$

By applying Kirchhoff's voltage law to the armature circuit

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + e_b \qquad \text{...(5.25)}$$

where e_b–back electromotive force (emf) given by

$$e_b = k_b \dot{\theta}_m$$

Hence
$$V_a = R_a i_a + L_a \frac{di_a}{dt} + k_b \dot{\theta}_m \qquad \text{...(5.26)}$$

Taking Laplace transform of equation (5.26)

$$V_a(s) = R_a i_a(s) + sL_a i_a(s) + sk_b \theta(s).$$

By simplifying and manipulating

$$i_a(s) = \frac{[V_a(s) - sk_b \theta_m(s)]}{[R_a + sL_a]} \qquad \text{...(5.27)}$$

From equations (5.24) and (5.27)

$$T(s) = k_a i_a(s)$$

$$T(s) = \frac{k_a [V_a(s) - sk_b \theta_m(s)]}{R_a + sL_a} \qquad \text{...(5.28)}$$

From equations (5.22) and (5.28)

$$s^2 I_{\text{eff}} \theta_m(s) + sC_{\text{eff}} \theta_m(s) = \frac{k_a [V_a(s) - sk_b \theta_m(s)]}{R_a + sL_a}$$

$$\theta_m(s) [s^2 I_{\text{eff}} + s C_{\text{eff}}] [R_a + sL_a] + sk_b \theta_m(s) = k_a V_a(s).$$

Transfer function is

Hence
$$\frac{\theta_m(s)}{V_a(s)} = \frac{k_a}{[s^2 I_{\text{eff}} + sC_{\text{eff}}][R_a + sL_a] + sk_b k_a} \qquad \text{...(5.29)}$$

Since the mechanical time constant is much higher than electrical time constant armature inductance can be comfortably neglected.

Hence
$$\frac{\theta_m(s)}{V_a(s)} = \frac{k_a}{s(s R_a I_{\text{eff}} + R_a C_{\text{eff}} + k_a k_b)} = \frac{K}{s(M_t s + 1)} \qquad \text{...(5.30)}$$

where
$$K = \frac{k_a}{R_a C_{\text{eff}} + k_a k_b} = \text{motor gain constant}$$

$$M_t = \frac{R_a I_{\text{eff}}}{R_a C_{\text{eff}} + k_a k_b} = \text{motor time constant}$$

Transfer function for the angular position of load,

$$\frac{\theta_L(s)}{V_a(s)} = \frac{NK}{s(M_t s + 1)} \qquad \text{...(5.31)}$$

Transfer function block diagram :

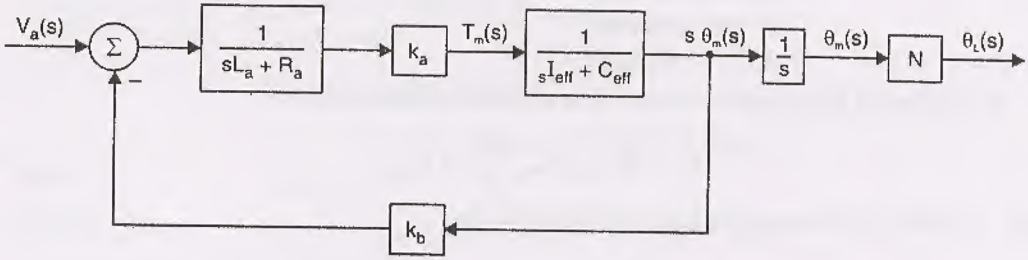


Fig. 5.11. Transfer Function of Single Joint Without Feedback.

Feedback Position Controller : (Proportional Controller)

The tracking of the desired angular displacement, θ_d by the actual angular displacement, θ by the motor is the function of the positional controller. The applied voltage to the motor is directly proportional to the error between θ_d and θ,

i.e.,
$$V_a = \frac{k_p E(t)}{N} = \frac{k_p[(\theta_1)_d - \theta_L]}{N} \qquad ...(5.32)$$

where k_p = position feedback gain.

It is possible to convert the open loop transfer function into a closed loop transfer function by unity negative feedback.

By applying Laplace transform to equation (5.32)

$$V_a(s) = \frac{k_p E(s)}{N} = \frac{k_p[(\theta_L)_d(s) - \theta_L(s)]}{N} \qquad ...(5.33)$$

By substitution of $V_a(s)$ into equation (5.30)

$$\frac{\theta_m(s)}{V_a(s)} = \frac{\theta_L(s)}{N\,V_a(s)} = \frac{\theta_L(s)}{E(s)}$$

$$\frac{\theta_L(s)}{E(s)} = \frac{k_p k_a}{s(sR_a I_{eff} + R_a C_{eff} + k_a k_b)} = G_p(s) \qquad ...(5.34)$$

For a unity feedback

$$\frac{\theta_L(s)}{(\theta_L)_d(s)} = \frac{G_p(s)}{1 + G_p(s)}$$

$$= \frac{k_p k_a}{[s^2 R_a I_{eff} + s(R_a C_{eff} + k_a k_b) + k_p k_a]} \qquad ...(5.35)$$

This function relates the actual angular displacement of the load with that of the desired angular displacement of the motor/load. The block diagram for unity feedback is as shown in Fig. 5.12.

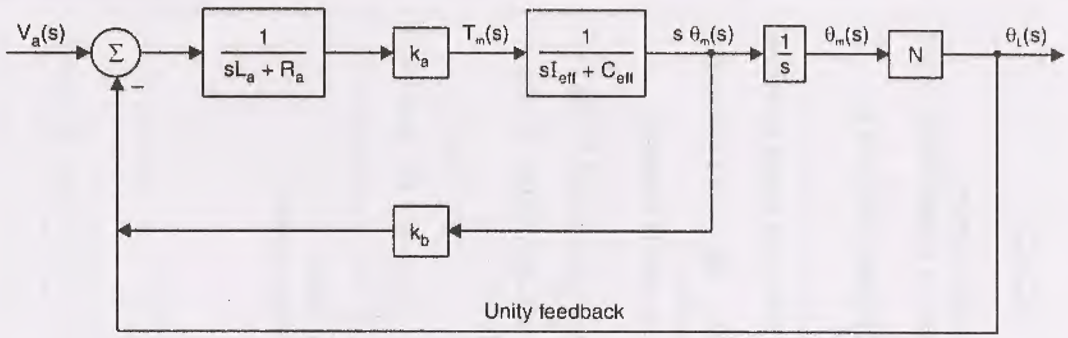


Fig. 5.12. Transfer Function of Single Joint with Feedback.

5.10. CONTROLLERS

The absolute control actions can be classified into four basic types : on-off controllers, proportional controllers, derivative controllers and integral type. By combining one or more control actions the following types of controllers are deviced.

(1) On-off controller

(2) Proportional controller

(3) Integral controller

(4) Proportional-Integral controller (PI)

(5) Proportional-Derivative controller (PD)

(6) Proportional-Integral-Derivative controller (PID).

In the controller's action it is the error that plays important role in deciding the actuating signal to manipulate the applied voltage of the armature. The error eliminating signal voltage may be proportional to the error in position, error in velocity or to the integral of error in position. The combination of more than one or two control action may be required to achieve the arm position within the specified limits.

- On-off controller : This provides two types of control-fully on or fully-off. Practically on-off controller is used when the error has to move through a range before occurrance of switch over.

$$\frac{V(s)}{E(s)} = M_1 \quad \text{for } e(t) > 0$$
$$= M_2 \quad \text{for } e(t) < 0$$

Table 5.8. Types of Controllers

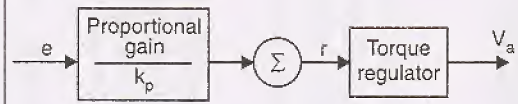
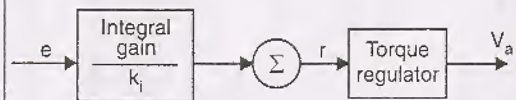
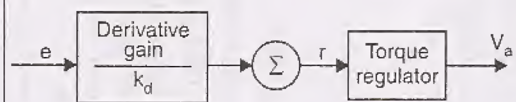
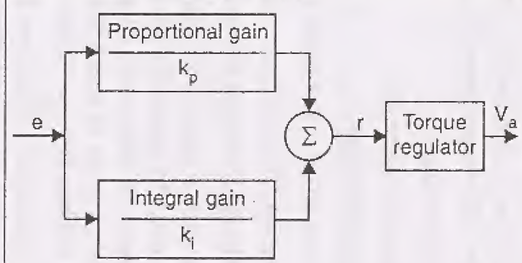
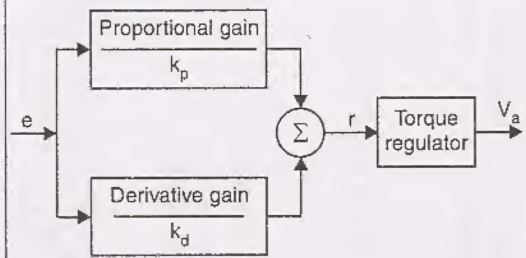
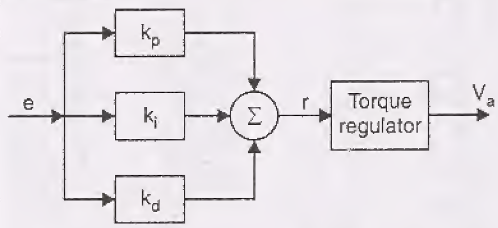
Error Function	*Laplace Function*	*Block Segment*	*Features*
$\mathbf{V_a(t) = k_p e(t)}$	$V(s) = k_p E(s)$ $\frac{V(s)}{E(s)} = k_p$		• Provides smoother control. • Control signal is proportional to the error. • Acts as an amplifier with gain k_p.
$\mathbf{V_a(t) = k_i \int e(t)\, dt}$	$\frac{V(s)}{E(s)} = \frac{k_i}{s}$		• For a large error the signal increases rapidly. • At zero error, o/p of the controller remains constant. • Used when the load is constant.
$\mathbf{V_a(t) = k_d \cdot \dot{e}(t)}$	$\frac{V(s)}{E(s)} = sk_d$		• Output signal proportional to the rate of change of error. • Anticipates change in error. • Faster response.
$\mathbf{V_a(t) = k_p e(t) + k_i \int e(t)\, dt}$	$\frac{V(s)}{E(s)} = k_p + \frac{k_i}{s}$		• Controller counteracts the load with proportional gain. • Slow response. • Provides zero error. • Combined control action.

Table 5.8. *Contd...*

Error Function	*Laplace Function*	*Block Segment*	*Features*
$\mathbf{V}_a(t) = k_p e(t) + k_d \dot{e}(t)$	$\frac{V(s)}{E(s)} = k_p + sk_d$		• Control signal proportional to rate of change of error, signal. • Faster response. • Rarely used alone.
$\mathbf{V}_a(t) = k_p e(t) + \int k_d e(dt) + k_d \dot{e}(t)$	$\frac{V(s)}{E(s)} = k_p + \frac{k_i}{s} + sk_d$		• Most general and commonly used controller. • Quick response. • Good control of system stability. • Low steady state error. • Computations are done on micro-computers.

5.11 THE PROPORTIONAL DERIVATIVE CONTROLLER

By introducing the derivative of the positional error it is possible to improve the system response and reduce the steady state error, which needs the addition of damping into the manipulator joint. This needs the measurement of velocity by the tachometer.

With the addition of this feedback term, the applied voltage to the joint motor is directly proportional to the position error and its derivative (velocity error)

i.e.,

$$V_a = k_p\,[(\theta_L)_d - \theta_L] + k_v\,[(\dot{\theta}_L)_d - \dot{\theta}_L]$$

$$= \frac{k_p\,E(t) + k_v\,\dot{E}(t)}{N} \qquad \text{...(5.36)}$$

where k_v = error derivative feedback gain.

By taking Laplace transform of equation (5.36)

$$V_a(s) = \frac{(k_p + sk_v)\,E(s)}{N} \qquad \text{...(5.37)}$$

Substituting $V_a(s)$ into equation (5.30)

$$\frac{\theta_L(s)}{E(s)} = \frac{k_a(k_p + sk_v)}{s(sR_a\,I_{eff} + R_a\,C_{eff} + k_a k_b)} = G_{PD}(s) \qquad \text{...(5.38)}$$

By unity negative feedback

$$\frac{\theta_L(s)}{(\theta_L)_d(s)} = \frac{G_{PD}(s)}{1 + G_{PD}(s)} \qquad \text{...(5.39)}$$

By manipulating equation (5.38)

$$\frac{\theta_L(s)}{(\theta_L)_d(s)} = \frac{k_a(k_p + sk_v)}{s^2R_a\,I_{eff} + s(R_a\,C_{eff} + k_a k_b + k_a k_v) + k_a k_p}$$

Refer Fig. 5.13 for the block diagram. ...(5.40)

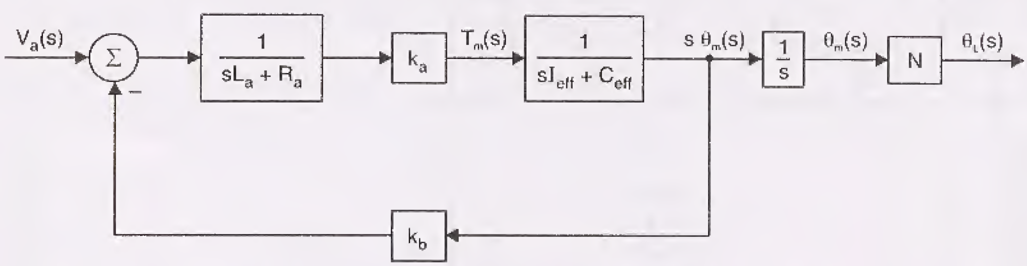


Fig. 5.13. Feedback Transfer Function for PD-controller.

• *Performance and stability measure*

To design the critically damped or overdamped system it is necessary to determine positional (k_p) and velocity (k_v) feed back gains.

The characteristic equation for a second order system is given by

$$s^2 + 2zw_n s + w_n^2 = 0 \qquad \text{...(5.41)}$$

Comparing this with equation (5.40)

$$w_n^2 = \frac{k_a k_p}{I_{eff}\ R_a} \qquad ...(5.42)$$

$$2zw_n = \frac{R_a . C_{eff} + k_a . k_b + k_a . k_v}{I_{eff} . R_a} \qquad ...(5.43)$$

- *Positional feed back gain*

$$k_p = \frac{w_n^2 . I_{eff} . R_a}{k_a} > 0 \qquad ...(5.44)$$

- *Damping ratio* (z)

$$z = \frac{R_a\ C_{eff} + k_a k_b + k_a k_v}{2\sqrt{k_a k_p I_{eff}\ R_a}} \geq 1 \qquad ...(5.45)$$

- *The velocity gain,* k_v

$$k_v \geq \frac{2\sqrt{k_a k_p I_{eff}\ R_a} - R_a\ C_{eff} - k_a k_b}{k_a} \qquad ...(5.46)$$

To prevent the resonace of the joint, it is suggested [2] that the undamped natural frequency may be set to no more than one-half of the structural resonant frequency, of the arm joint

$$w_n \leq 0.5\ w_r \qquad ...(5.47)$$

Substituting this in equation (5.42), the positional feedback gain is given by

$$0 < k_p \leq \frac{w_r^2\ I_{eff}\ R_a}{4\ k_a} \qquad ...(5.47a)$$

If at a pre-assumed moment of inertia I_0 the resonant frequency is w_0, then

$$w_r^2 . I_{eff} = w_0\ I_0$$

or

$$w_r = w_0 \sqrt{\frac{I_0}{I_{eff}}}\ , \qquad ...(5.48)$$

and the equation (5.47a) takes the form

$$k_p \leq \frac{w_0^2 I_0\ R_a}{4\ k_a} \qquad ...(5.49)$$

The velocity feedback gain is,

$$k_v \geq \frac{R_a w_o \sqrt{I_o I_{eff}} - R_a\ C_{eff} - k_a k_b}{k_a} \qquad ...(5.50)$$

PROBLEMS

Example 5.1. *A certain robot manipulator has a mechanical joint described by the differential equation specifying the position of the output link as a function of time,*

$$2.2\frac{d^2y}{dt^2} + 17.6\frac{dy}{dt} + 35.2y = X(t)$$

where y gives the output response and X is the function describing the force applied.

(*a*) *Write the characteristic equation.*

(b) What are the roots of the characteristic equation ?

(c) Determine the type of response.

(d) Write the transfer function.

(e) Write the natural frequency and the resonant frequency.

(f) Calculate the damping ratio.

Sol. Given the differential equation for the robot joint,

$$2.2\frac{d^2y}{dt^2} + 17.6\frac{dy}{dt} + 35.2y = X(t)$$

(*a*) The characteristic equation by Laplace transform

$$2.2\, s^2\, y(s) + 17.6\, s\, y(s) + 35.2\, y(s) = X(s)$$

$$2.2s^2 + 17.6\, s + 35.2 = 0. \qquad ...(i)$$

(*b*) Roots of equation (*i*)

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

From equation (*i*)

$$a = 2.2 \quad b = 17.6 \quad c = 35.2$$

Hence,
$$S_{1,2} = \frac{-17.6 \pm \sqrt{(17.6)^2 - 4(2.2)(35.2)}}{2 \times 2.2}$$

$$S_1 = S_2 = -4.$$

(*c*) Type of response $b^2 - 4ac = 0$

Hence the system is critically damped.

(*d*) Transfer function

$$(2.2s^2 + 17.6\, s + 35.2)\, Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{(2.2s^2 + 17.6s + 35.2)}. \qquad ...(ii)$$

(*e*) Natural frequency

By comparing equation (*ii*) with

$$\frac{1}{s^2 + 2zw_n s + w_n^2}$$

$$w_n^2 = \frac{35.2}{2.2}$$

$$\therefore \qquad w_n = \sqrt{\frac{35.2}{2.2}} = 4 \text{ rad/sec}$$

Resonant frequency

$$w_{res} = 2w_n = 8 \text{ rad/sec.}$$

(*f*) Damping ratio

$$2zw_n = \frac{17.6}{2.2}$$

$$z = \frac{8}{2w_n} = \frac{8}{2 \times 4} = 1.$$

Hence the system his critically damped transient response.

Example 5.2. *Given the following set of equations, write the block diagram and the transfer function by the block diagram reduction technique, by assuming X as input and Y as output.*

$$W = \frac{dz}{dt} + 10\,z\ ;\ \frac{dy}{dt} - y = z\ ;\ W = x + 8y.$$

Sol. By Laplace Transform, taken on the given differential equations

$$W(s) = sz(s) + 10z(s) = (s + 10)\, z\,(s) \quad \ldots(a)$$

$$Z(s) = sy(s) - y(s) = (s - 1)\, y(s) \quad \ldots(b)$$

$$X(s) = w(s) - 8y(s) \quad \ldots(c)$$

By substituting W(s) and Z(s) into equation (c)

$$X(s) = (s + 10)\, z\,(s) - 8y(s)$$

$$= (s + 10)\,(s - 1)\, y(s) - 8y(s)$$

Now

$$\frac{Y(s)}{X(s)} = \frac{1}{(s+10)\,(s-1)-8} \text{ is the transfer function.}$$

Block diagram representation,

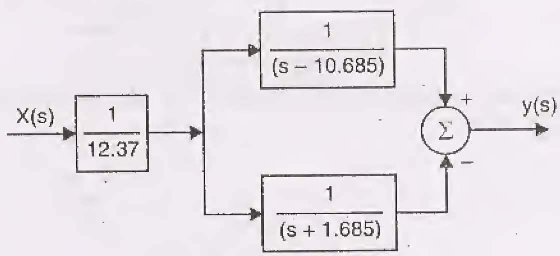


Fig. 5.14

Method 1 :

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + 9s - 18} = \frac{1}{(s - 10.685)\,(s + 1.685)}$$

By partial fraction

$$= \frac{1}{12.37(s - 10.685)} - \frac{1}{12.37(s + 1.685)}$$

$$= \frac{1}{12.37}\left[\frac{1}{(s - 10.685)} - \frac{1}{(s + 1.685)}\right]$$

and $\dfrac{X(s)}{s^2 + 9s - 18} = y(s)$ and $\dfrac{Y(s)}{X(s)} = \dfrac{1}{(s^2 + 9s - 18)}$

Method 2 :

$$\frac{X(s)}{Y(s)} = (s + 10)\,(s - 1) - 8$$

But we have from (a), (b) and (c)

$$8y(s) = W(s) - X(s)$$

$$W(s) = (s + 10)\, z(s)$$

$$z(s) = (s - 1)\, y(s)$$

Block diagram

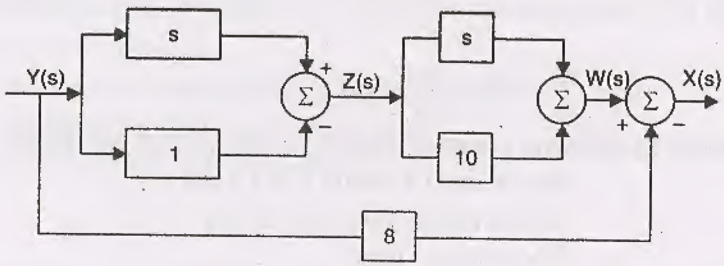
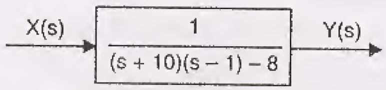


Fig. 5.15

Hence $$\frac{X(s)}{Y(s)} = (s + 10)(s - 1) - 8$$

or $$\frac{Y(s)}{X(s)} = \frac{1}{(s+10)(s-1)-8}$$

Reduced Block diagram



Example 5.3. *Develop a state space model for the single link (single-axis) robot manipulator, shown in Fig. 5.16.*

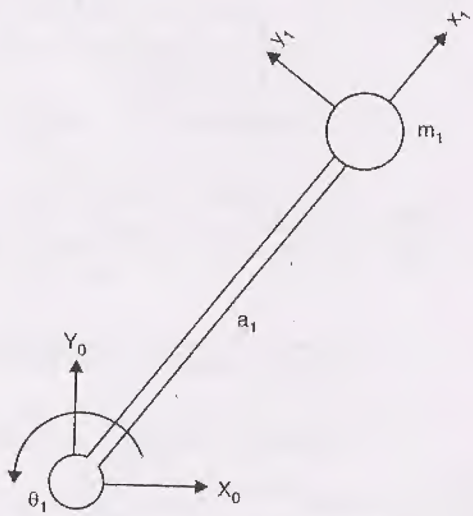


Fig. 5.16. Single Link Manipulator.

Sol. Dynamic equation of motion

$$T = m_1 a_1^2 \ddot{q} + m_1 g a_1 \cos\theta_1 + c(\dot{q}) \qquad ...(5.51)$$

The state equation $\dot{q} = V$

$$\dot{V} = a = \frac{T - m_1 g a_1 \cos\theta_1 - c(V)}{m_1 a_1^2} \qquad ...(5.52)$$

Tool tip position vector

$$p = [a_1 \cos \theta_1, a_1 \sin \theta_1 , 0]^T$$

The kinematic output equation

$$y = [a_1 \cos \theta_1, a_1 \sin \theta_1, 0, 0, 0, 1]^T.$$

Example 5.4. *The differential equation describing a joint in a robotic manipulator is as below.*

$$2.5 \frac{d^2 y}{dt^2} + 20 \frac{dy}{dt} + 40y = 0.$$

Determine the following :

(a) Delay time

(b) Settling time

(c) Peak over shoot.

Sol. The characteristic equation from the differential equation

$$2.5 s^2 + 20s + 40 = 0.$$

This compares with $s^2 + (2zw_n)s + w_n^2 = 0$.

where $$w_n^2 = \frac{40}{2.5}$$

$$w_n = \sqrt{16} = 4 \text{ rad/sec.}$$

and $$2zw_n = \frac{20}{2.5} = 8$$

$$z = \frac{8}{2w_n} = 1.$$

From Table (5.4)

(*a*) Delay time,

$$T_d = \frac{1.0 + 0.6z + 0.15z^2}{w_n} = \frac{1.75}{8} = 0.218 \text{ sec.}$$

(*b*) Settling time,

$$T_s < \frac{4.7}{zw_n} = \frac{4.7}{(1)(4)} = 1.175 \text{ sec.}$$

(*c*) Peak overshoot, $$M_0 = \exp\left(\frac{-z\pi}{\sqrt{1-z^2}}\right)$$

$$= \exp\left[\frac{-(1)\pi}{\sqrt{1-1}}\right] = 0.$$

Example 5.5. *For a spring-mass-damper system with stiffness $k = 5$, mass, $m = 1$, and the damping co-efficient $c = 4$. The system is known to passes unmodelled resonance at $w_{res} = 6.0$ rad/sec. Determine the velocity gain and the positional gain to critically damp the system with resonably high stiffness.*

Sol. From equation (5.47)

$$w_n = \frac{1}{2} w_{res} = \frac{6}{2} = 3 \text{ rad/sec.}$$

Using the equation (5.15)

$$w_n = \sqrt{\frac{k_p k_m}{J}}$$

But, $k_m = 1$ and $J = m = 1$

$$k_p = w_n^2 = 9$$

From equation (5.16)

$$z = \frac{c + k_v k_m}{2(Jk_p k_m)}$$

and

$$k_v = \frac{2(Jk_p k_m)z - c}{k_m}$$

$$2zw_n = c$$

and

$$z = \frac{c}{2w_n}$$

But $c = 4$ and $z = \frac{4}{2 \times 6} = \frac{1}{3}$

Hence

$$k_v = \frac{[2(1 \times 9 \times 1)\frac{1}{3} - 4]}{(1)} = 2.0$$

Ans. Hence velocity gain, $k_v = 2$

Positional gain, $k_p = 9$.

Example 5.6. *A certain potentiometer is used as a feed back device for link rotation of the industrial robot arm.*

The excitation voltage of the potentiometer is B volts and the total wiper travel of the potentiometer is 300°. If the potentiometer is connected directly without reduction

Determine

(a) voltage constant, k_p

(b) the output voltage for a rotation of 35° of the wiper.

Sol. (*a*) k_p voltage constant $V_a(t) = k_p, \theta_a$.

$$k_p = \left(\frac{V_a(t)}{\theta_a}\right)_{\max} = \frac{(8) \times 360}{300 \times 2\pi} = 1.527 \text{ V/rad.}$$

(*b*) Output voltage for $\theta = 35°$

$$V_0 = k_p \,.\, \theta_0 = 1.527 \left(\frac{\pi}{180}\right) .\, 35 = 0.933 \text{ volts.}$$

Ans. (*a*) $k_p = 1.527$ V/rad.

(*b*) $V_0 = 0.933$ V.

Example 5.7. *A dc tachometer attached actuator drives the twisting joint of a robotic manipulator, with a provision for velocity feedback. The actuator can drive the joint with maximum angular speed of 0.8 rad/sec. If the tachometer constant is 9 V/rad/sec. What is the maximum output voltage of the device ? The ratio of rotational speed of the device to that of the joint is 2 and the joint rotates at a angular speed of 30°/sec. Find the output voltage of the tachometer for this case.*

Sol. Data : $(\theta_a)_{\max} = 0.8$ rad/sec.

$$(\theta_t)_{max} = N \cdot (\theta_a)_{max} = 1.6 \text{ rad/sec (as N = 2)}$$

$$(\theta_a)_i = 30°/\text{sec} = \left(\frac{\pi}{180}\right) \times 30° = 0.523 \text{ rad/sec}$$

$$(\theta_t)_i = N(\theta_a)_i = 2(0.523) = 1.047 \text{ rad/sec.}$$

$$k_v = 9 \text{ V/rad/sec.}$$

(*a*) The maximum voltage of the tachometer

$$(V_a)_{max} = k_v \cdot (\theta_a)_{max} = 9 \times 1.6 = 14.4 \text{ volts.}$$

(*b*) The desired output voltage

$$(V_a)_i = k_v \cdot (\theta_a)_i = 9 \times 1.047 = 9.42 \text{ volts.}$$

Ans. **(*a*) The maximum voltage of the tachometer = 14.4 V**

(*b*) The required voltage for 30°/sec = 9.42 volts.

EXERCISE

5.1. Describe any four controllers used in Robotic systems mentioning their respective transfer function. *(VTU–Jan./Feb. 2003)*

5.2. Describe with transfer function any four controllers used in robotic system. *(VTU–Jan./Feb. 2004)*

5.3. Describe any four controllers used in robotic system and mention their respective transfer function. *(VTU–May/June 2004)*

5.4. List different types of controllers employed for Robot control. Explain any two of them.

5.5. Draw block diagram and obtain the transfer function that corresponds to spring-mass-damper system suspended from a fixed wall.

5.6. Explain clearly the four responses of a second order system.

5.7. Explain the transient response for a second order system.

5.8. Explain the control concept of a robotic manipulator system.

5.9. Explain the different phases of control of robot motion.

5.10. Write and explain general block diagram of robot control system.

5.11. List the design parameters used in the control of robot manipulator joint. Explain any three of them.

5.12. Define and explain the following :

(*a*) The delay time

(*b*) The setting time

(*c*) The peak overshoot.

5.13. With a block diagram explain the modeling of the control system with the help of general dynamic equation.

5.14. Explain the joint transfer function with rate feed back with respect to the control of a single link manipulator.

5.15. With the block diagram and transfer function explain the following :

(*a*) Proportional control

(*b*) Derivative control

(*c*) Integral control.

5.16. Explain the advantages of the following controllers :

(*a*) PD controller

(*b*) PID controller.

5.17. Explain the use of damping factor and the natural frequency in deciding the response of a robotic joint described by a differential equation.

5.18. Develop a model for the transfer function of a dc motor driving a robot joint.

5.19. A mechanical joint a certain robot manipulator has its motion described by the differential equation, $3\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 4y = X(t)$,

where y specifies the output and $X(t)$ describing the forcing function.

(*a*) write down the characteristic equation and the transfer function.

(*b*) Find out the roots of the characteristic equation.

(*c*) Determine the type of response.

(*d*) Compute the natural frequency and the resonant frequency.

(*e*) Calculate the damping ratio.

5.20. Given the following set of equations, write down the block diagram assuming X as the input and Y as the output

$$\frac{dz}{dt} + 5z = w$$

$$\frac{dy}{dt} - 3y = z$$

$$x + 8y = w$$

(*a*) Determine the transfer function by block diagram reduction.

(*b*) Determine the roots of the characteristic equation and the type of response.

5.21. The differential equation specifying the joint motion in a robotic manipulator is

$$3\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 5y = 0$$

Determine the following:

(*a*) The setting time

(*b*) The delay time

(*c*) Peak overshoot.

5.22. For a spring-mass-damper system with stiffness $k = 4$, mass $m = 2$ and the damping co-efficient $c = 4$. The system is known to passes unmodeled response at $w_{resonant} = 8.0$ rad/sec. Determine the velocity gain, k_v and the positional gain k_p of the critically damped system with reasonably high stiffness.

5.23. A twisting joint of a robotic manipulator driven by an actuator with a dc tachometer giving velocity feed back. If the tachometer constant is 7.5 volts/rad/sec and the maximum output voltage is 15 V, what is the maximum angular speed of the joint possible. The joint rotates with half the speed of the actuator through gear reduction. If the joint rotates at a angular speed of 35°/sec. Find the output voltage for this case of operation.

6 Trajectory Planning

6.1 INTRODUCTION

The performance of task manipulation by the tool or the end-effector needs the solution of the problem of trajectory or the path plan, with the use of kinematic formulation, which has been discussed in chapter 3. The mathematical models of kinematic formulation give the description of tool position and orientation within the work-envelope.

The trajectory or the path refers to a time history of position, velocity and acceleration of all possible movements of the manipulator. The internal functions and the computing systems represent the path or trajectory generation. The manipulator hand moves through a series of position and orientation starting from a initial location to a final location generating in its path a well defined space curve.

The path planning module receives certain set of inputs in various forms described as follows.

- Path specification : is one of the approaches for generating trajectory points, with the use of a polynomial analytical function.
- Path constraints : The intermediate points generated have to be connected to make the path continuous and smooth so that the position and the time derivatives like velocity and accelerations in the path are realistic and traversable.
- Dynamic constraints : Involve the usage of joint torque and force control in the process of path generation and control. The dynamics becomes prominent in the high speed operations of the manipulator. The optimization of the path has to be done with the consideration of dynamics of the manipulator.

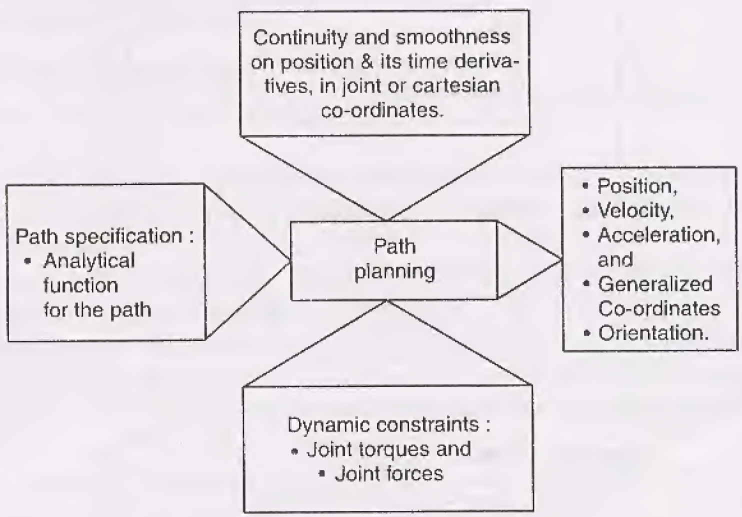


Fig. 6.1. Path Planning Block Diagram.

The outputs of the path planner are the position and orientation of the tool at various points in the path. The joint variable space vectors like velocity, acceleration and the generalized co-ordinates give the insight into the path outputs, for the control of the robot arm. The Fig. 6.1 gives the block diagram of path planner.

6.2 PATH CONTROL MODES

The movement of the arm of a robot to be controlled has the following schemes to be considered under two possible constraints : (1) The trajectory constraints and (2) The obstacle (hurdle) constraints.

Mode 1 : When there is no obstacle and there is no constraint on the path to be followed the positional control is sufficient.

Mode 2 : When there is certain obstacle and no path constraint the positional control is to be employed with hurdle detection and avoidance.

Mode 3 : When there is no obstacle in the path and tool has to move in certain given path, the off line path planning with on-line path tracking has to be carried out.

Mode 4 : The presence of a hurdle with a definite constraint on path needs off-line collission free path planning coupled with on-line path tracking.

The above four modes of path control are schematically illustrated by the Fig. 6.2.

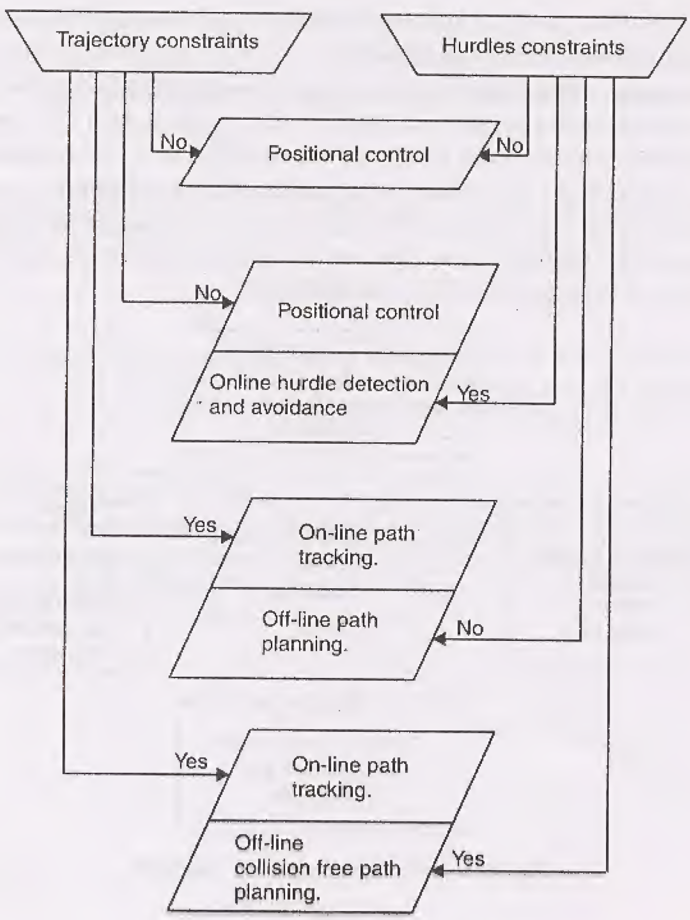


Fig. 6.2. Path Control Schemes.

6.3 GENERAL CONSIDERATION IN TRAJECTORY PLANNING

• Frame Basis

It is advantageous to describe and generate path by specifying robot motions in terms of tool frame with respect to the base frame.

• Specification of Path

The decoupled motion description of the tool frame from a particular robot or end effector results in module formation. This helps in usage of the same description to different manipulators or different sizes of tool.

• Movements of Manipulator

The manipulator motion is identified and relative to the initial and final position of the tool frame with respect to the base frame.

• Sequence of Intermediate Points

This involves in specifying sequence of intermediate points between the end positions of the tool frame with respect to station frame. The tool has to pass through these intermediate points (Position and orientations) describing the path. The joining of these points gives the space curve or the spatial constraint.

• Elapse Time

The specifying of elapsed time between intermediate points in the description of the path gives temporal attributes of the manipulator motion.

• Smooth Traverse

To describe the motion the path has to be described by a well defined smooth function which is also continuous with two of its (first and second) time derivatives being continuous. Wear and tear or vibration of the mechanism are attributed by rough and jerky motion. By constraining the spatial and temporal qualities of the path between the points it is possible to ensure the smoothness of operation.

• Schemes of Path Generation

There are two methods of path generation : (1) The joint space scheme, (2) Cartesian space scheme.

(1) Joint Space Scheme : In this the time history of all joint variables and their first two time derivatives are used for planning the motion of the manipulator, Refer Fig. 6.3 for the flow-chart of this scheme.

(2) Cartesian Space Scheme : In this method the manipulator's hand position, velocity and acceleration time history are used in planning and using these joint space variables are computed. Fig. 6.4 shows the sub-routine flow-chart for this type of scheme.

In the above two schemes, the manipulator joint solution has to be computed and updated for every control interval. The four constraints that are to be considered as input to the trajectory planning function are,

- Computation of trajectory point directly using expressions for a given time interval.
- Generation of intermediate points, in the path.
- Incorporating continuity and smoothness in the joint positions and its time derivatives.
- The minimization of wandering motions of the joint.

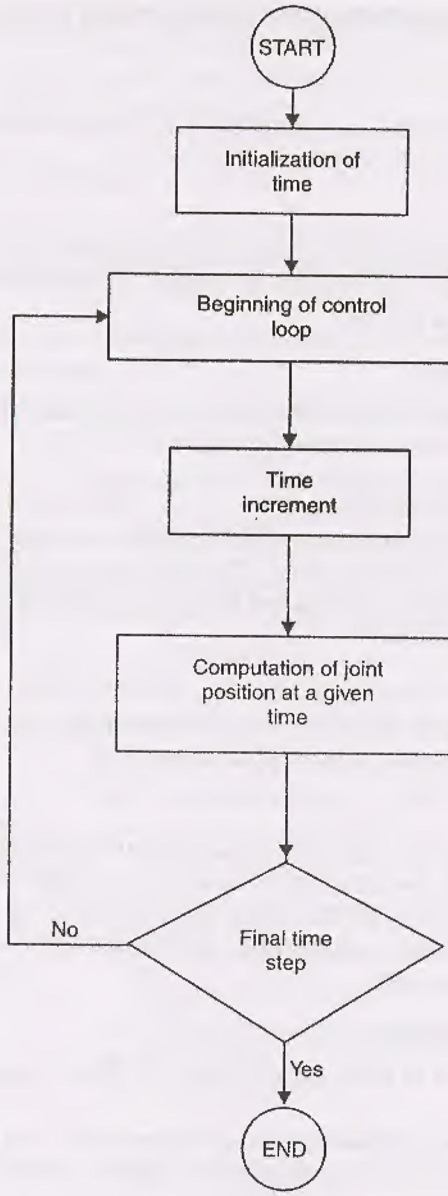


Fig. 6.3. Flow Chart.

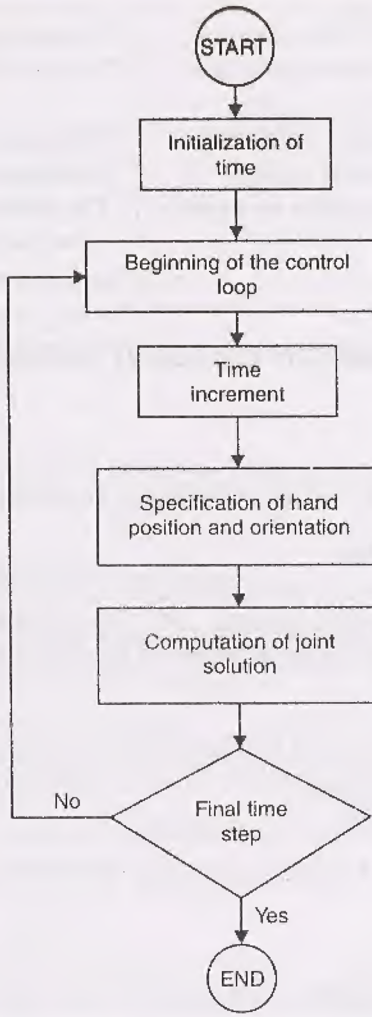


Fig. 6.4. Flow Chart.

Table 6.1. Joint-Space Versus Cartesian Space

Type of Scheme	*Advantages*	*Disadvantages*
1. Joint space scheme	• Planning of trajectory from control variables in direct terms. • Real time planning of trajectory. • Ease of generating trajectory planning. • Uses low-degree polynomials to interpolate joint hinge points. • Computationally faster. • Dealing of the manipulator dynamic constraints is relatively easy.	• Difficulty in determining the location of the joints, links and hand during motions. • Obstacle avoidance in the path is very difficult. • Less accurate along the cartesian path.

2. Cartesian space scheme	• The method is straight forward. • The assured accuracy along the straight line path. • Easy determination of link and had locations during motion. • Obstacles can be easily avoidable.	• Computationally intensive. • Longer control intervals. • Mapping the hand co-ordinates into joint co-ordinates is not properly defined. • The problem in optimization because of mixed constraint specifications in path and dynamic variables.

6.4 GENERAL CONSIDERATION OF JOINT INTERPOLATED TRAJECTORY

• Hand Motion Direction

In the object pick-up, the motion of the hand has to be directed away from the position of the object, to prevent the crashing of the end-effector into the object support structure.

• Admissible Departure Motion

The departure motion requires the specification of lift-off point along a normal vector to the surface of the object in the initial position, and passing of the hand through this position. The speed control can be achieved by specifying the time required to reach this position.

• Set-down Motion

This requirement for this type of motion is same as that for departure motion from the lift-off point, to achieve the correct approach direction and control.

• Positions in the Trajectory

In the path to be generated by the hand there are four positions : (1) Initial (2) Lift-off. (3) Set-down. (4) Final.

• Constraints of Position

(*a*) Velocity and acceleration of initial position (normally assumed zero).

(*b*) Continuous motion at intermediate points in the lift-off-position.

(*c*) Continuous motion at the set down point.

(*d*) Velocity and acceleration specified for the final position (normally assumed zero).

• Work-Space Limitation

The physical and geometric limitation of each joint determine the extremity of the joint trajectories.

• Time Constraint

The time specification for the initial and final trajectory segments depends on (1) rate of approach (2) a constant fixed based on the actuator characteristics.

For the segments of mid-trajectory the time is specified based on maximum velocity and acceleration of the joints.

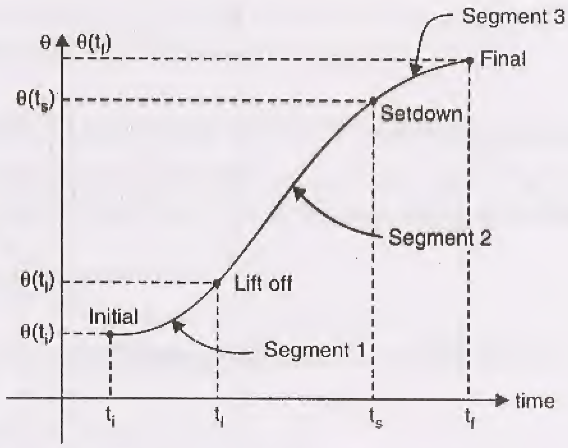


Fig. 6.5. Joint Trajectory Positions.

6.5 TRAJECTORY GENERATION PLANNING

Joint Space	*Trajectory Planning*	*Cartesian Trajectory Planning*
Low-order Planning	*Higher-order Planning*	
• By third-order polynomial. $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$	• 4–3–4 trajectory – Segment 1 : 4^{th} degree polynomial. – Segment 2 : 3^{rd} degree polynomial. – Segment 3 : 4^{th} degree polynomial.	• Segmented differential motion : Smooth straight line transformation between initial and final position by fine segmentation of the path.
• By fifth order polynomial. $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$.	• 3–5–3 trajectory – Segment 1 : 3^{rd} degree polynomial. – Segment 2 : 5^{th} degree polynomial. – Segment 3 : 3^{rd} degree polynomial. • 5-Cubic trajectory 5 segments of 3^{rd} degree.	• Motion by T2R motion : The transformation between initial and final positions is divided into one translation and two rotations. • Motion by 1T1R motion : The transformation between initial and final position is decomposed into a translation and a rotation about an arbitrary axis.

6.6 LINEAR PATH WITH PARABOLIC BLEND

The simplest path for the end effector is along straight line. But at the deviations the EE can not traverse a straight line path. Hence a blend which is parabolic is assumed. The Fig. 6.6 shows the theoretical linear and the actual blended path trajectory.

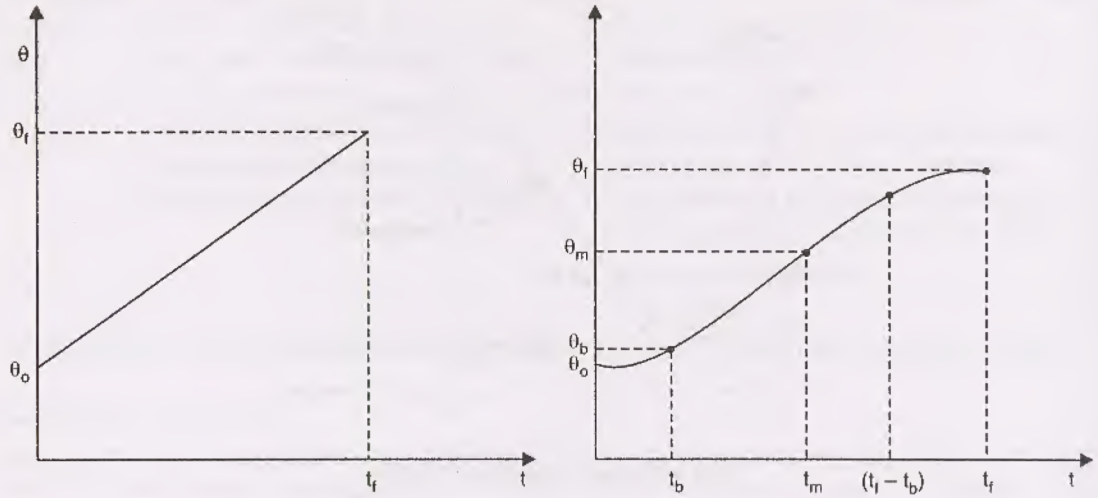


Fig. 6.6. Straight Path and Parabolic Blend.

The velocity at the end blend region must be equal and continuous.

So,

$$\ddot{\theta} \,.\, t_b = \frac{\theta_m - \theta_b}{t_m - t_b} \qquad \text{...(6.1)}$$

$$\ddot{\theta} .\, t_b t_m - \ddot{\theta}\, t_b^{\,2} = \theta_m - \theta_b$$

Substituting $\quad t_m = \dfrac{t}{2}$

$$\ddot{\theta}\, t_b . \frac{t}{2} - \ddot{\theta}\, t_b^{\,2} = \theta_m - \theta_b$$

$$\ddot{\theta}\, t_b . t - 2\,\ddot{\theta}\, t_b^{\,2} = 2(\theta_m - \theta_b).$$

By Law of motion at the blend

$$\theta_b = \theta_0 + \frac{1}{2}\ddot{\theta}\, t_b^{\,2} \qquad \text{...(6.2)}$$

$$\ddot{\theta}\, t_b . t - 2\,\ddot{\theta}\, t_b^{\,2} = 2\theta_m - 2\left(\theta_0 + \frac{1}{2}\ddot{\theta}\, t_b^{\,2}\right)$$

$$\ddot{\theta}\, t_b\, t - \ddot{\theta}\, t_b^{\,2} = \theta_f - 2\theta_0, \text{ as } \theta_f = 2\theta_m.$$

or $\quad \ddot{\theta} t_b^2 - \ddot{\theta} t_b t + (\theta_f - 2\theta_0) = 0. \qquad \text{...(6.3)}$

The equation (6.3) is a quadratic with t_b, hence,

$$t_b = \frac{-(-\ddot{\theta}t) \pm \sqrt{(\ddot{\theta}t)^2 - 4\ddot{\theta}\,(\theta_f - 2\theta_0)}}{2\ddot{\theta}} \qquad \text{...(6.4)}$$

The critical constraint gives acceleration,

$$\ddot{\theta} = \frac{4(\theta_f - 2\theta_0)}{t^2} \qquad \text{...(6.5)}$$

With this acceleration there will not be any linear portion in the path. With increase in acceleration the parabolic blend shrinks to a shorter portion.

6.7 TRAJECTORY PLANNING WITH 3rd ORDER POLYNOMIAL

To describe the path traced by the hand of the robot a polynomial of the following form is assumed and four set of boundary conditions are imposed to solve the problem to arrive at the joint variables.

- The polynomial :

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad \text{...(6.6)}$$

- The boundary conditions are imposed on equation (6.6)

at $t = t_i = 0$; $\theta(t) = \theta(t_i) = a_0$.

at $t = t_f$; $\theta(t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$

Time derivatives,

at $t = t_i = 0$; $\dot{\theta}(t_i) = 0, a_1 = 0.$

at $t = t_f$; $\dot{\theta}(t_f) = 0.$

By differentiating equation (6.6) twice w.r.t. 't'

$$\dot{\theta}(t) = \frac{d\theta}{dt} = a_1 t 2a_2 t + 3a_3 t^2$$

and

$$\ddot{\theta}(t) = \frac{d^2\theta}{dt^2} = 2a_2 + 6a_3 t$$

By knowing starting and finish joint angle and intermediate time the joint angles for various positions can be obtained. The respective joint velocities and the accelerations can also be computed.

6.8 TRAJECTORY PLANNING WITH 5th ORDER POLYNOMIAL

A polynomial with order 5, is used to describe the trajectory of the end-effector. Along with initial and final, positions and velocities, it is also possible to specify the starting and final accelerations.

Given polynomial,

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \quad \text{...(6.7)}$$

By differentiation

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \quad \text{...(6.8)}$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 \quad \text{...(6.9)}$$

Boundary conditions

- for position,

at $t = t_i = 0$; $\theta(t) = \theta_i = a_0$

at $t = t_f$; $\theta(t) = \theta_f$

- for velocity,

at $t = t_i = 0$; $\dot{\theta}(t) = \dot{\theta}_i = a_1$

at $t = t_f$; $\dot{\theta}(t) = \dot{\theta}_f$

- for acceleration,

at $t = t_i = 0\ ;\ \ddot{\theta}(t) = \ddot{\theta}_i = 2a_2$

at $t = t_f\ ;\ \ddot{\theta}(t) = \ddot{\theta}_f$

6.9 4-3-4 TRAJECTORY CALCULATION

This type of trajectory has three segments. The first segment is from the initial position to lift of position, which is described by a fourth degree polynomial. The second segment is third degree polynomial between lift-off and set down positions. The third segment is between set down to final position, specified by again a fourth degree polynomial.

• First Segment

The polynomial is $\theta(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3 + a_{14}t^4$ ...(6.10)

The time derivatives $\ddot{\theta}(t) = 2a_{12} + 6a_{13}t + 12a_{14}t^2$ ...(6.11)

$\dot{\theta}(t) = a_{11} + 2a_{12}t + 3a_{13}t^2 + 4a_{14}t^3$ ...(6.12)

Boundary conditions

For $t = 0,\ \theta(t) = \theta_0$

$\theta_0 = a_{10}$ ...(Given data)

and $\dot{\theta}(t) = \dot{\theta}_0 = a_{11}$

Also $\ddot{\theta}(t) = \ddot{\theta}_0 = 2a_{12}$

or $a_{12} = \dfrac{\ddot{\theta}_0}{2}$

Hence the polynomial take the form

$$\theta(t) = \theta_0 + \dot{\theta}_0 t + \frac{\ddot{\theta}_0}{2}t^2 + a_{13}t^3 + a_{14}t^4 \qquad \text{...(6.13)}$$

At the end of the first segment the acceleration and the velocity have to be continuous irrespective of the position through the same intermediate point.

Assuming, $t = 1$

$$\dot{\theta}_1 = \dot{\theta}(1) = a_{11} + 2a_{12} + 3a_{13} + 4a_{14}$$

$$\ddot{\theta}_2 = 2a_{12} + 6a_{13} + 12a_{14}.$$

• Second Trajectory Segment

The polynomial is of third degree,

$$\theta(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3 \qquad \text{...(6.14)}$$

By differentiation $\dot{\theta}(t) = a_{21} + 2a_{22}t + 3a_{23}t^2$ ...(6.15)

$\ddot{\theta}(t) = 2a_{22} + 6a_{23}t$ ...(6.16)

Boundary condition : At lift off

$t = 0$

$$a_{20} = \theta_2(0)\ ;\ a_{21} = \dot{\theta}_2(0)\ ;\ a_{22} = \frac{\ddot{\theta}_2(0)}{2}$$

Boundary condition at the end of second segment for continuous velocity and acceleration,

$$t = 1$$

$$\theta_2(1) = a_{20} + a_{21} + a_{22} + a_{23}$$

$$\dot{\theta}_2(1) = a_{21} + 2a_{22} + 3a_{23}$$

$$\ddot{\theta}_2(1) = 2a_{22} + 6a_{23}$$

The polynomial takes the form

$$\theta(t) = \theta_2(0) + \dot{\theta}_2(0).\, t + \frac{\ddot{\theta}_2(0)}{2}.\, t^2 + a_{23}t^3 \qquad ...(6.17)$$

- **Last Trajectory Segment**

The polynomial is of fourth degree.

$$\theta_n(t) = a_{n0} + a_{n1}t + a_{n2}t^2 + a_{n3}t^3 + a_{n4}t^4 \qquad ...(6.18)$$

At $t = 0$, $\theta_n(0) = a_{n0}$

$$\dot{\theta}_n(0) = a_{n1}$$

$$\ddot{\theta}_n(0) = 2a_{n2}$$

or $a_{n2} = \dfrac{\ddot{\theta}_n(0)}{2}$

At $t = 1$, $\dot{\theta}_n(1) = a_{n1} + 2a_{n2} + 3a_{n3} + 4a_{n4}$

$$\ddot{\theta}_n(t) = 2a_{n2} + 6a_{n3} + 12a_{n4}$$

The polynomial takes the form

$$\theta_n(t) = \theta_n(0) + \dot{\theta}_n(0)t + \frac{\ddot{\theta}_n(0)}{2}\, t^2 + a_{n3}t^3 + a_{n4}t^4. \qquad ...(6.19)$$

PROBLEMS

Example 6.1. *One of the joint of a articulated robot has to traverse from an initial angle of 20° to a final angle of 84° in 4 seconds. Using a third degree polynomial calculate the joint angle at 1, 2 and 3 second.*

Sol. The given third degree polynomial is

$$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

The time derivatives given by

$$\dot{\theta}(t) = a_1 + 2a_2t + 3a_3t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3t$$

At $t = t_i = 0$, $\theta(t_i) = a_0 = 20$

$$\dot{\theta}(t_i) = a_1 = 0 \text{ (starting velocity zero)}$$

At $t = 4 = t_f$, $\theta(t_f) = a_0 + a_1(4) + a_2(4)^2 + a_3(4)^3 = 84$

$$\dot{\theta}(t_f) = a_1 + 2a_2(4) + 3a_3(4)^2$$

The simultaneous equations are

$$16a_2 + 64a_3 = 64$$

$$8a_2 + 48a_3 = 0$$

The solution of the simultaneous equation yields

$$a_2 = -12 \text{ and } a_3 = -2$$

The polynomial is
$$\theta(t) = 20 - 12t^2 - 2t^3$$
$$\dot{\theta}(t) = -24t - 6t^2$$

At the given times

t	1	2	3
θ	6°	–44°	–142°

Example 6.2. *The path traced by a joint of a robot manipulator is described by the fifth degree polynomial. The joint has to start from an initial angle of 10° to 20°. The starting acceleration and the ending deceleration 2 deg / sec². The velocities being zero, find the equation of motion for the joint. The range is covered in 2 seconds.*

Sol. Fifth degree polynomial describing the joint trajectory, given as,

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$
$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

For the boundary condition,

$$t = t_i = 0$$
$$\theta(t_i) = a_0 = 10$$
$$\dot{\theta}(t_i) = a_1 = 0$$
$$\ddot{\theta}(t_i) = 2a_2 = 2 \text{ and } a_2 = 1.$$

The motion equation takes the form

$$\theta(t) = 10 + t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
$$\dot{\theta}(t) = 2t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$
$$\ddot{\theta}(t) = 2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

For the boundary condition,

$$t = t_f = 2$$
$$\theta(2) = 20 = 10 + 4 + 8a_3 + 16a_4 + 32a_5$$
$$\dot{\theta}(2) = 0 = 4 + 12a_3 + 32a_4 + 80a_5$$
$$\ddot{\theta}(2) = -2 = 2 + 12a_3 + 48a_4 + 160a_5$$

The solution of the simultaneous equations yields

$$a_3 = 13 \,;\, a_4 = -10 \,;\, a_5 = 2$$

The equations of motion are

$$\theta(t) = 10 + t^2 + 13t^3 - 10t^4 + 2t^5$$
$$\dot{\theta}(t) = 2t + 39t^2 - 40t^3 + 10t^4$$
$$\ddot{\theta}(t) = 2 + 78t - 120t^2 + 40t^3.$$

The angle, velocity and acceleration at $t = 1$

$$\theta(1) = 16°$$

$$\dot{\theta}(1) = 11 \text{ deg/sec}$$

$$\ddot{\theta}(1) = 0 \text{ deg/sec}^2.$$

Example 6.3. *The revolute joint of an articulated PTP robot traverses from an initial position of 10° to 40° final position, in 3 seconds. Assuming a third degree polynomial and start-off acceleration 3 deg / sec, determine the deceleration at the end of 3 seconds.*

Sol. The third degree polynomial expressed by

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

The boundary condition, $t = t_1 = 0$ gives

$$a_0 = 10 \; ; a_1 = 0 \; ; \text{and } a_2 = 1.5$$

The polynomial is $\theta(t) = 10 + 1.5t^2 + a_3 t^3$

$$\theta(3) = 40 = 10 + 14.4 + 27a_3$$

$$a_3 = \frac{15.6}{27}$$

$$\ddot{\theta}(3) = 3 + 6 \times \frac{15.6}{27} \,.\, (3)$$

$$= 13.4 \text{ deg/sec.}$$

The deceleration is equal to 13.4 deg/sec.

Example 6.4. *A joint of a robot manipulator traverses from an initial position of 25° to a final position of 75° in 5 seconds. Assuming a fifth degree polynomial and a starting acceleration of 4 deg / sec². Determine the acceleration at the end of 5 seconds.*

Sol. Assumed polynomial,

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Taking the time derivative,

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

At $t = t_i = 0$, gives $\theta(t_i) = 25$, $\ddot{\theta}(t_i) = 4$, $\dot{\theta}(t_i) = 0$

Hence, $a_0 = 25 \; ; a_1 = 0$ and $a_2 = 2$

Using the b.c. $t = t_f = 5$, $\theta(t_f) = 75°$ $\dot{\theta}(t_f) = 0$

Hence, $75 = 25 + 2t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

or

$$125a_3 + (5)^4 a_4 + (5)^5 a_5 = 0 \qquad ...(a)$$

$$20 + 75a_3 + 500\, a_4 + (5)^5 a_5 = 0 \qquad ...(b).$$

From relations (a) and (b)

$$50\, a_3 + 125\, a_4 = 20$$

By trial and error, $a_3 = 25.4$ and $a_4 = -10.$

Substitution of a_3 and a_4 into relation (a)

$$a_5 = \frac{123}{125}$$

Hence, $$\ddot{\theta}(5) = 2(2) + 6(25.4)(5) + 12(-10)(25) + 20\left(\frac{123}{125}\right)125$$

$$= 126 \text{ deg/sec}^2$$

The acceleration is 126 deg/sec^2.

Example 6.5. *A robot manipulator with single link moves from an initial position of – 5° to a final position of 80° in four seconds with a smooth stop. Compute for a linear trajectory and parabolic blend find the acceleration and plot as a function of time.*

Sol. The expression for motion given by equation (6.3)

$$\ddot{\theta}\, t_b^{\,2} - \ddot{\theta}\, t_b \,.\, t + (\theta_f - 2\theta_0) = 0.$$

Given are the data $\theta_f = 80°$, $\theta_0 = -5°$.

Assuming t values and substituting in the following expression.

$$\ddot{\theta} = \frac{(\theta_f - 2\theta_0)}{t_b(t - t_b)} = \frac{90}{t_b(t - t_b)}$$

Table 6.2

t_b	t	$\ddot{\theta}$	$\dot{\theta} = 90/\log [t_b(t-t_b)]$
$t_b = 1$	2	90° deg/sec^2	∞
	3	45 deg/sec^2	130 deg/sec
	4	30 deg/sec^2	82 deg/sec
$t_b = 1.5$	2	120 deg/sec	– 313 deg/sec
	3	40 deg/sec	111 deg/sec
	4	24.0 deg/sec.	68 deg/sec.

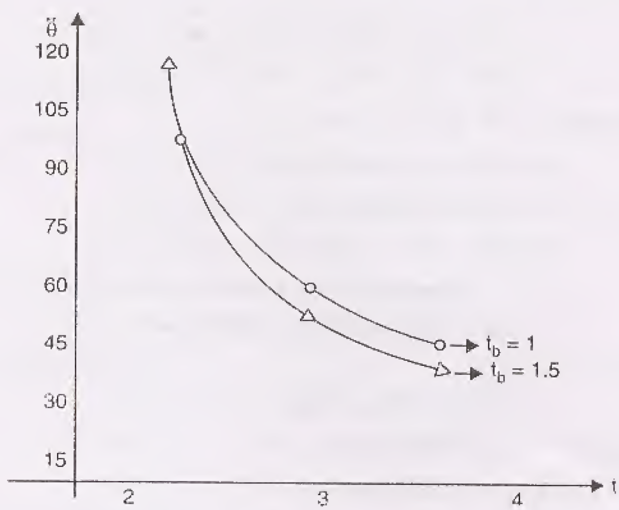


Fig. 6.7. Acceleration Plot.

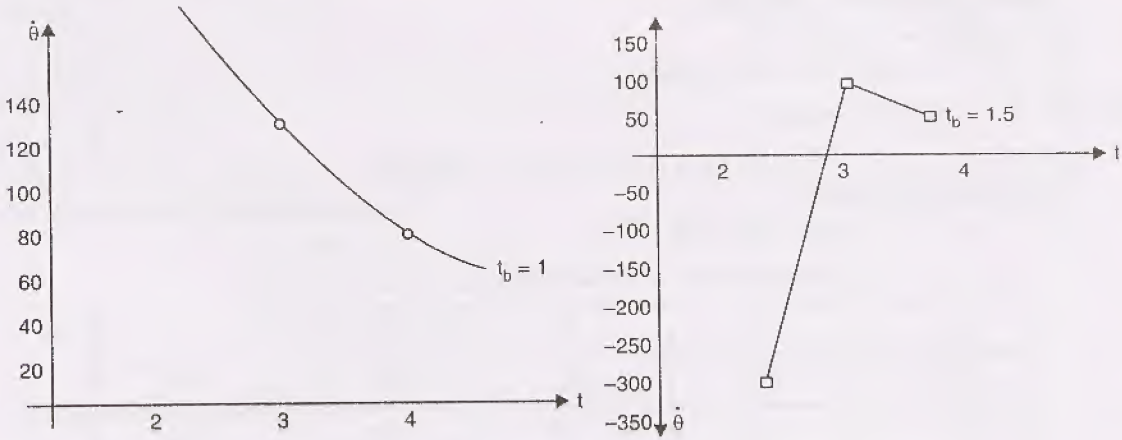
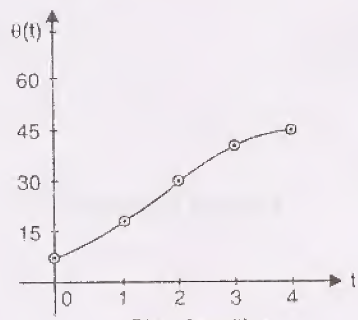


Fig. 6.8. Velocity Plot.

Example 6.6. *A joint in a PTP robot, which rotates from an initial angle of 5° to a final angle of 65° in 5 sec with a constant maximum velocity of 12 deg/sec. Determine the position of the joint in 1, 2, 3, 4 secs, and plot the results.*

Sol. The polynomial to be selected for the constant velocity of the joint is

$$\theta(t) = a_0 + a_1 t$$



Plot of position

The first time derivative is $\dot{\theta}(t) = a_1$

The imposed boundary conditions are

at $t = 0, \theta = 5°$

and $t = 5, \theta = 65°$

So,

$$\theta(0) = a_0 + a_1(0) = 5°$$
$$\theta(5) = a_0 + a_1(5) = 65$$

Hence,

$$a_1 = \frac{60}{5} = 12$$

and the polynomial is

$$\theta(t) = 5 + 12t$$

t	1	2	3	4
$\theta(t)$	17°	29°	41°	53°

Example 6.7. *A PTP robot with a revolute joint moving with velocity of 15 deg/sec, traverses from an initial position of 15° to a final position of 75 deg/sec. Determine the position and velocity at the end of 1, 2, 3, 4 seconds. The range of initial and final position is covered in 5 secs with a finite acceleration of 6 deg/sec^2.*

Sol. The motion of the path of the joint rotating with finite acceleration is described by the polynomial,

$$\theta(t) = a_0 + a_1 t + a_2 t^2$$

The time derivatives are

$$\dot{\theta}(t) = a_1 + 2a_2 t$$
$$\ddot{\theta}(t) = 2a_2$$

Applying boundary conditions,

At $t = 0, \theta = \theta_0 = a_0 = 15°$

At $t = 5, \theta = \theta_5 = 15° + 5a_1 + 25a_2$

and at $t = 0, \dot{\theta}_0 = 15$ deg/sec.

So, $\dot{\theta}_0 = 15 = a_1, \ddot{\theta} = 6 = 2a_2 \Rightarrow a_2 = 3$

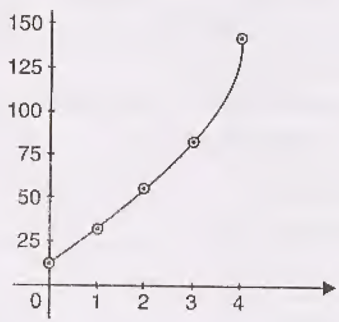
Now the polynomial is

$$\theta(t) = 15 + 15t + 3t^2$$

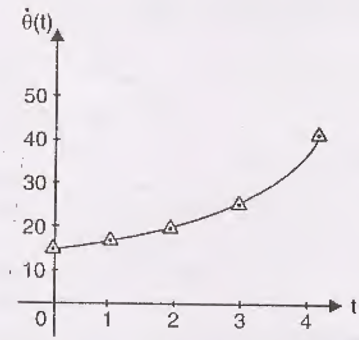
$$\dot{\theta}(t) = 15 + 2 \times 3t = 15 + 6t$$

t_{sec}	1	2	3	4
$\theta(t)$	33°	57°	87°	133°
$\dot{\theta}(t)$	21	27	33	49

Plot of Position :



Plot of Velocity :



Example 6.8. *For a robot it is given that initial angle at time 0 sec is 15° and the final position given by 75° after 3 sec. Determine expression for position, velocity and acceleration of joint and plot the result, using cubic polynomial.*

Sol. Let the cubic polynomial be

$$\theta(t) = a_1 + a_2t + a_3t^2 + a_4t^3$$

at $t = 0, \theta = 15°$

Hence, $a_1 = 15$

By differentiating $\theta(t)$ w.r.t. t

$$\dot{\theta}(t) = a_2 + 2a_3t + 3a_4t^2$$

at $t = 0, \dot{\theta} = 0$

Hence $\quad a_2 = 0.$

and *at* $t = 3$, $\dot{\theta} = 0$

$$6a_3 + 27a_4 = 0 \qquad \text{...}(a)$$

At $t = 3$, $\theta = 75°$

i.e., $$75 = 15 + 9a_3 + 27a_4 \qquad \text{...}(b)$$

Solving (a) and (b) simultaneously

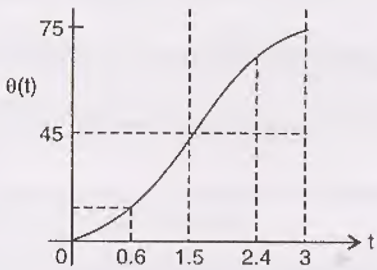
$$a_3 = 20, \quad a_4 = -4.44$$

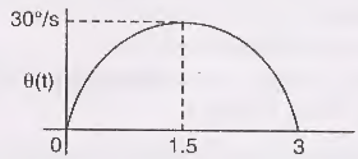
$$\theta(t) = 15 + 20t^2 - 4.44t^3$$

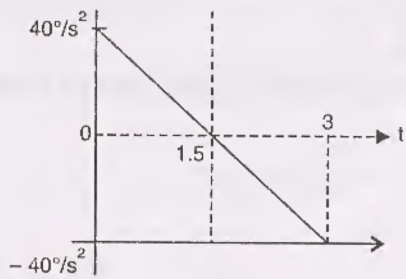
$$\dot{\theta}(t) = 40t - 13.32t^2$$

$$\ddot{\theta}(t) = 40 - 26.64t$$

Position plot







EXERCISE

6.1. Explain trajectory planning and show how trajectory planning is done in case of PTP (point-to-point) robot having constant maximum velocity and finite acceleration and deceleration. *(VTU–May / June 2004)*

6.2. Discuss trajectory planning with respect to PTP robot considering modified constant velocity motion of the joint. *(VTU–Jan. / Feb. 2004)*

6.3. Define trajectory planning and demonstrate the same for a PTP robot considering a modified uniform velocity of motion. *(VTU–Jan. / Feb. 2003)*

6.4. What are the various forms of input required to be provided as constraints to the robot path planning ?

6.5. Describe different path control modes in robotics.

6.6. Discuss the advantages and disadvantages between joint space scheme and cartesian-space schemes.

6.7. Enumerate trajectory generation polynomial types.

6.8. Explain the parameters involved in the path planning with 3rd degree polynomial.

7 Robot Sensors and Vision

7.1 INTRODUCTION

The interaction of the robot with the environment set-ups needs mechanisms known as sensors, that can perform the following functions :

- Motion control variables, detection.
- Robot guidance without obstruction.
- Object identification tasks.
- Handling the objects.

The sensors that provide the informations like joint position, velocity and acceleration are known as internal state sensors.

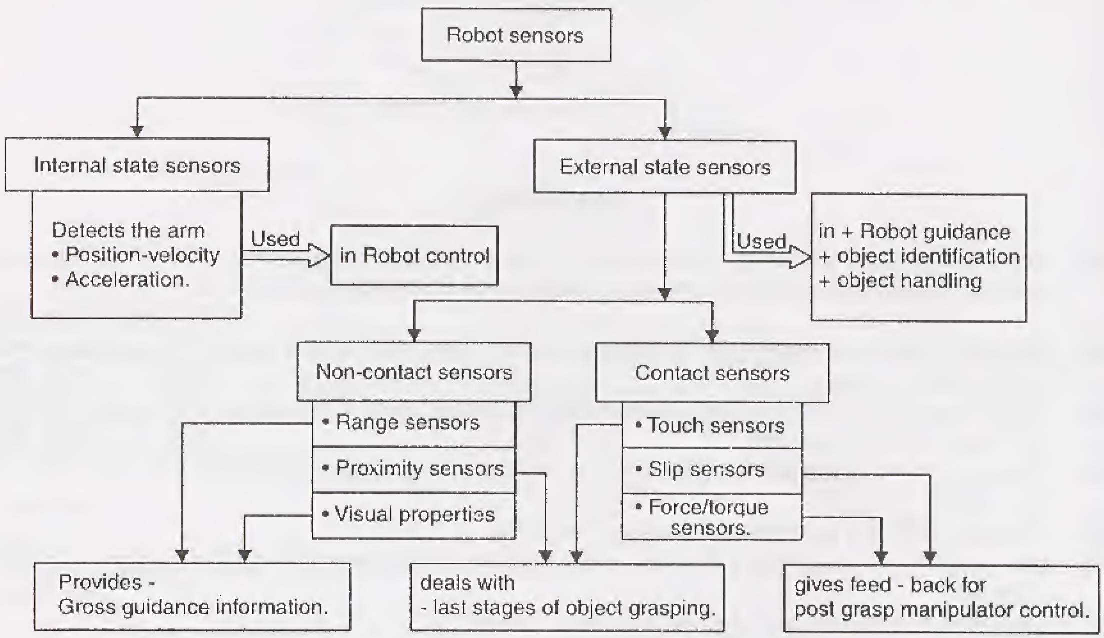
The robots are being guided by the help of vision and range sensors that are known as non-contact external state sensors.

The task of object identification is done by proximity and touch sensors known as contact type external state sensors.

The informations of object handling are supplied as a feedback from force and torque sensors termed as contact type internal state sensors.

The brief classification of sensors based on functions is given in section 7.2.

7.2 CLASSIFICATION OF SENSORS AND THEIR FUNCTIONS



7.3 TOUCH SENSORS

The touch sensors gather the informations established by the contact between the parts to be handled and the fingers in the manipulator end effectors. The signals of touch informations are useful in

- locating the objects.
- recognising the object type.
- Force and torque control needed for task manipulation.

The Types of Touch Sensors are :

1. *Binary sensors* detect the existence of the object to be handled. For example, micro-switches and limit switches.
2. *Analog sensors* produce proportional output signal for the force exerted locally. For example, a code wheel with a plunger.

A useful application of binary sensors is to use it on a robot engaged in contact inspection of the parts. A robot with six degree of freedom can provide higher manuarability compared to three axis co-ordinate measuring machine.

7.4 BINARY SENSORS

The devices that deliver sensing signal by contact at two gripping points are termed the binary sensors. The fingers as shown in Fig. 7.1. accommodate the binary sensors. The contact with the parts results in deflection and this information is sufficient to determine the presence of the object between the fingers. The proper grasping and manipulation of the object in the work envelope can be easily achieved through centering of the fingers assisted by the information given by binary sensors.

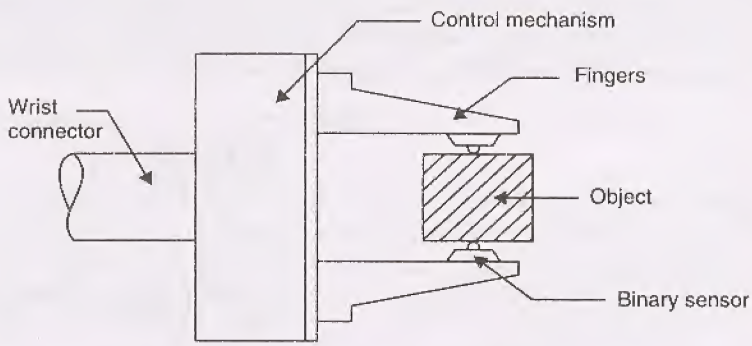


Fig. 7.1. Hand with Binary Sensors.

7.5 ANALOG SENSORS

This type of sensors are featured by spring actuated plunger connected to a code wheel. The deflection of the plunger rod by the action of contact force, results in rotation of the wheel which gives an output proportional to the sensors force. The schematic arrangement of the analog sensor is as shown in Fig. 7.2.

If k is the spring rate and δ is the deflection of the plunger recorded, the force of contact is given by

$$F = k\,\delta. \qquad ...(7.1)$$

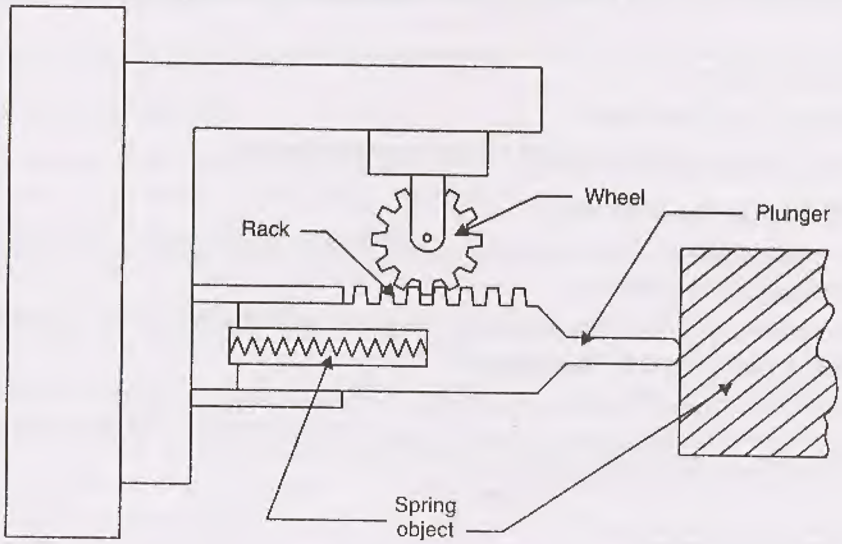


Fig. 7.2. Analog Sensor.

7.6 TACTILE SENSORS

An array of touch sensors arranged systematically to provide information about the contact of the fingers with the object is called the tactile sensors. The special tactile sensors also provide additional informations like shape, size and the type of material of the objects.

Each element in an array (tactile sensor) there are three functional parts : A plunger, a LED and a light sensing device. The schematic is as shown in Fig. 7.3. The movement of the plunger opens/blocks the LED, and the light sensor gives output signal accordingly.

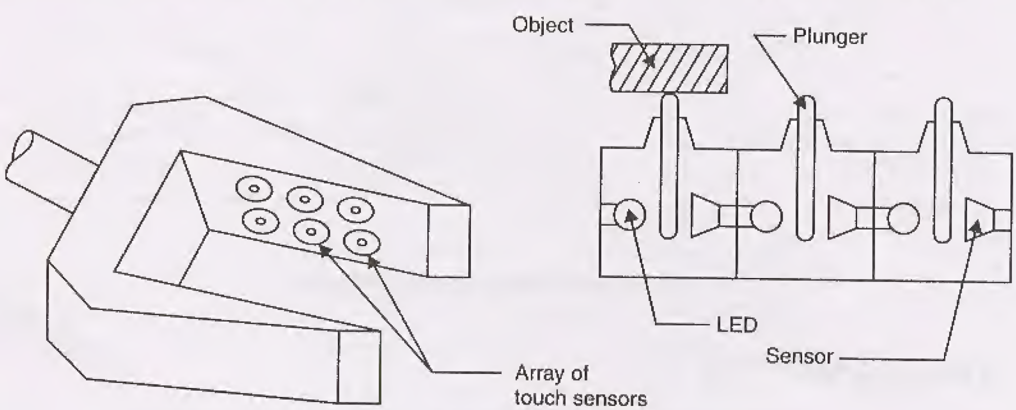


Fig. 7.3. Tactile Sensors.

7.7 DESIRABLE FEATURES FOR SENSORS AND TRANSDUCERS

Features	*Functions*
• Precision	→ Should be as high as possible. → Deviation in measurement reading should be minimum.
• Accuracy	→ Should be as high as possible. → Error between sensed value and actual value should approach zero.
• Speed of response	→ Time taken to respond to variation should be minimum. → Response to be instaneous.
• Operating range	→ Range operating to be wide. → Accuracy over the range to be acceptable.
• Reliability	→ The life to be high. → Frequent failures are not acceptable.
• Calibration	→ Should be easy to calibrate. → Drift to be minimum. → Should take less time to calibrate without much trouble.
• Cost and ease	→ The cost of purchase should be low. → The installation and operation should be easy and less costly.

7.8 PROXIMITY SENSORS

The output of the proximity sensors gives an indication of the presence of an object with in the vicinity job operation. In robotics these sensors are used to generate information of object grasping and obstacle avoidance. This section deals with some of the important proximity sensors used in robotics.

• Inductive Sensors

* *Principle*

The ferromagnetic material brought close to this type of sensor results in change in position of the flux lines of the permanent magnet leading to change in inductance of the coil. The induced current pulse in the coil with change in amplitude and shape is proportional to rate of change of flux line in magnet.

* *Construction*

The proximity inductive sensor basically consists of a wound coil located infront of a permanent magnet encased inside a rugged housing. The leads from the coil, embedded in resin is connected to the display through a connector. The schematic is as shown in Fig. 7.4.

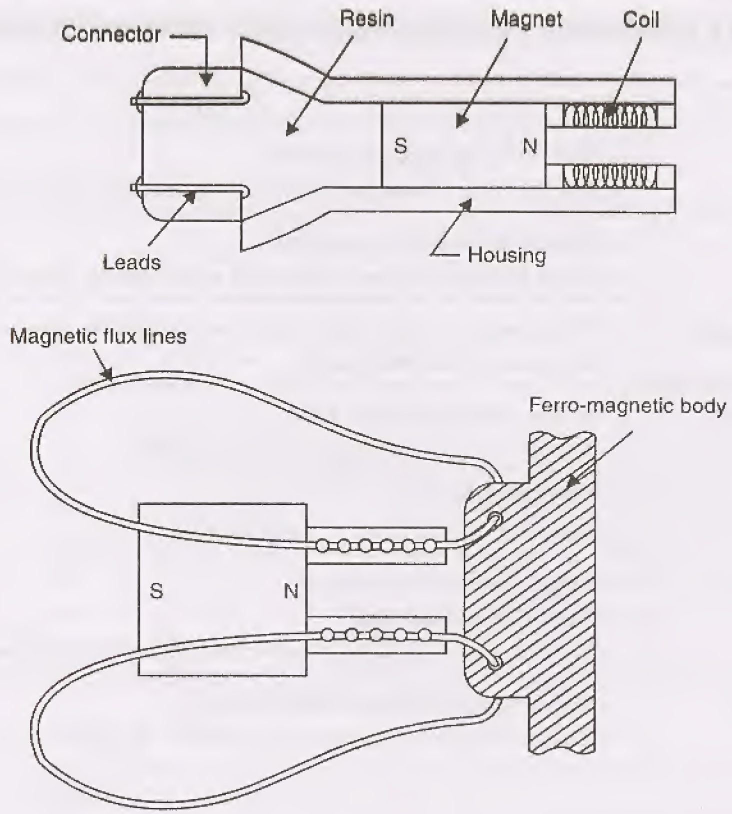


Fig. 7.4. Inductive Sensor.

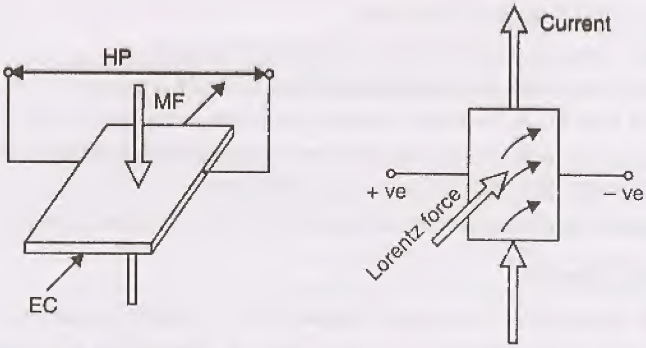
• Hall-Effect Sensors

* *Principle*

Hall-Effect deals with the voltage between the two points in a conductor which changes by the near field of the magnetised or ferromagnetic material. The sensor experiences a weakened magnetic field in the close proximity of a ferromagnetic materials, due to the bending of the flux lines of the magnet through approaching object.

E.R. Hall in 1879 discovered Hall Effect, which states that "A beam of charged particles passing through a magnetic field experiences a force that deflect the beam from the straight line path".

Electrons (negative charged particles) are made to pass through a plate rectangular in shape and a magnetic field is applied at right angle to the plane of plate as shown in Fig. 7.5(*a*). The electrons are deflected towards one side of the plate making that side negatively charged and other side positively charged. The force due to applied magnetic field is known as Lorentz force. The mechanism of deflection is governed by the balance of Lorentz force and force on the beam of electrons.



HP = Hall Potential ; MF = Magnetic Field ; EC = Electric Current

Fig. 7.5 (*a*) Hall Effect Principle.

* *Construction*

A sensor element is stationed between the poles of a horse shoe magnet constructed inside a container. The principle of operation is as depicted in Fig. 7.5 (*b*).

The decrease in the strength of the magnetic field resulting due to the proximity of the object field reduces the voltage across the sensor. The sensor gives binary output for the decision making devices of control for further actions. The silicon makes the ideal selection for a semiconductor interms of size, strength and capacity to electrical interference prevention.

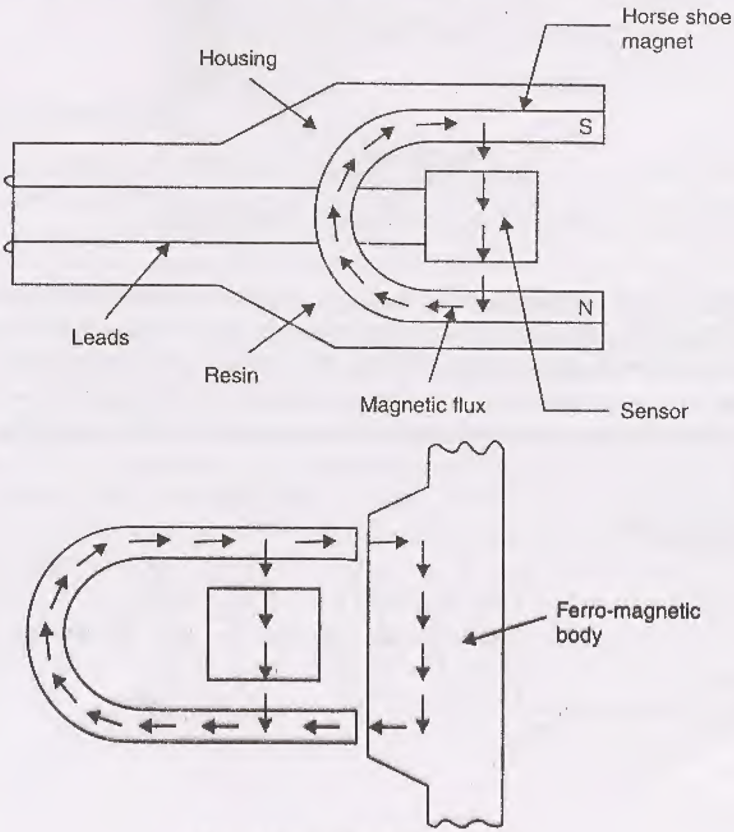


Fig. 7.5 (*b*) Hall-Effect Sensor.

- Advantages of Hall Effect Sensors :
  - ✓ They can operate as switches at high frequency.
  - ✓ They cost less than electromechanical devices.
  - ✓ They are free from contact bounce problem.
  - ✓ They can be used under severe environmental service conditions as they are immune to environmental contaminations.
  - ✓ They can be used as proximity, position and displacement sensors.

Ultrasonic Proximity Sensor

The previously discussed proximity sensors are useful for detection of ferro-magnetic matter only. If the robot has to handle other type of materials ultrasonic sensors find the application.

* *Construction*

The main part in this type of sensor is the transducer which can act both as transmitter and receiver. The sensor is covered by a resin block which protects from dust and humidity. For the acoustic damping, absorber material is provide as shown in Fig. 7.6 (*a*). Finally a metallic housing gives general protection.

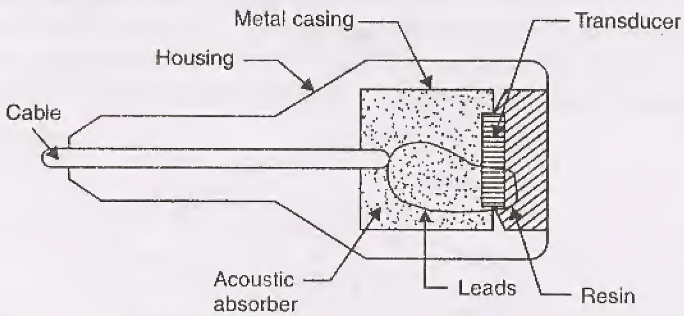


Fig. 7.6 (*a*) Ultrasonic Proximity Sensor.

* *Operation*

The acoustic waves emitted by the sensors reach the object and get reflected and the receiver sensors the waves to generate the information about the presence of the object. This type of operation is the echomode, type. When the sensor acts only as the transmitter the waves get blocked by the presence of the object and the receiver gets no signal. This type is known as opposed mode. The echomode type of operation is as shown in Fig. 7.6 (*b*).

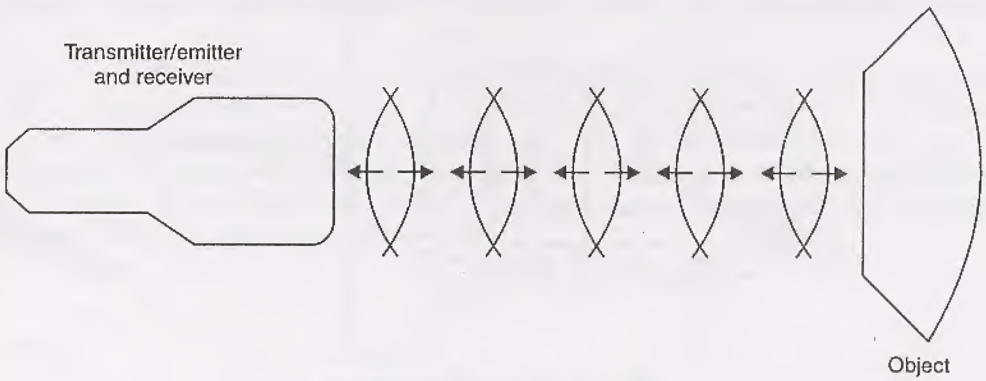


Fig. 7.6 (*b*) Echomode Operation.

Optical Sensors

Optical Sensors are similar to ultrasonic sensors. The proximity of the object is detected by the action of the travelling light wave as it propogates from the transmitter and reflected by the object towards the receiver.

The Fig. 7.7 shows the constructional details of the optical sensor. The light emitted by a diode is focussed by the transmitter lens, on to the object surface. The reflected light waves travel back and received by the solid-state photo diode, through a receiver lens. When the object is within the range of the sensor it is possible to detect the presense of the receiver. The range is defined by the position and orientation of the object and the focal length of the sensor lenses.

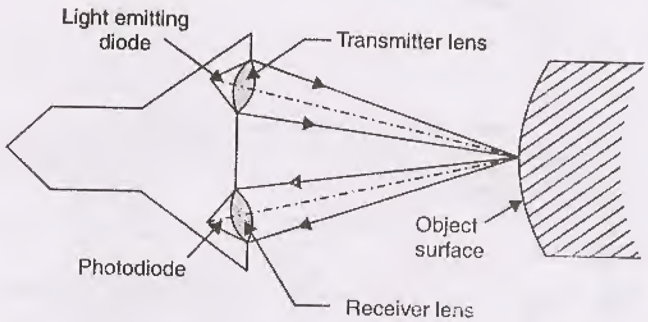


Fig. 7.7. Optical Proximity Sensor.

7.9 RANGE SENSORS

The distance between the object and the robot hand is measured using the range sensors within it is range of operation. The calculation of the distance is by visual processing. Range sensors find use in robot navigation and avoidance of the obstacles in the path. The exact location and the general shape characteristics of the part in the work evelope of the robot is done by special applications for the range sensors. There are several approaches like, triangulation method, structured lighting approach and time-of-flight range finders etc. In these cases the source of illumination can be light-source, laser beam or based on ultrasonics.

* *Triangulation Method*

This is the simplest of the techniques, which is easily demonstrated in the Fig. 7.8. The object is swept over by a narrow beam of sharp light. The sensor focussed on a small spot of the object surface detects the reflected beam of light. If 'θ' is the angle made by the illuminating source and 'b' is the distance between source and the sensor, the distance 'd' of the sensor on the robot is given as

$$d = b \cdot \tan \theta \qquad \ldots(7.2)$$

The distance '*d*' can be easily transformed into 3D-co-ordinates

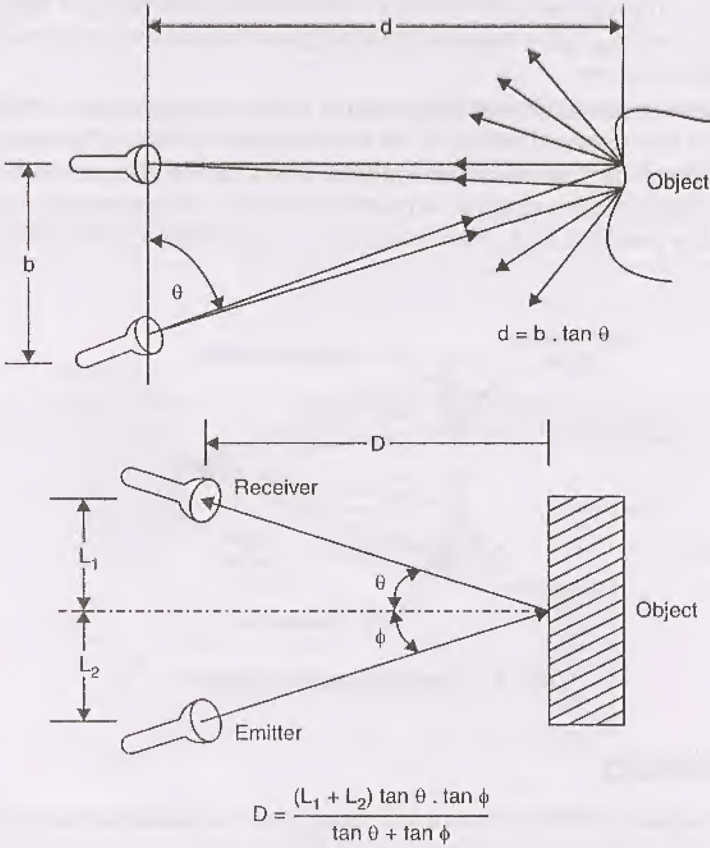


Fig. 7.8. Triangulation Method of Range Sensing.

7.10 FORCE AND TORQUE SENSORS

The wrist force sensor shown in Fig. 7.9 is used to measure the force and torque induced on the wrist of the robotic manipulator. They can also be used to measure the joint forces. Typical force/torque sensor work on the strain gauge principle. The change in the resistance of the electrical strain gauges effected by the strain due to change in force induced is a measure of force and torque.

The construction of the sensor has got a disc housing support and a deflection bar. The strain gauges are mounted on the six faces of the deflection bar. The force on the wrist is transformed into measurable deflections or displacements at the wrist.

A balanced wheatstone bridge is used to arrange the four resistance. The galvanometer connected between X and Y with equal potential shows zero deflection when there is no force exerted. The force on the wrist changes the resistance in any one arm, which results in current flow and leads to the movement of the galvanometer needle. The change in resistance is given by

$$\frac{R_1}{R_4} = \frac{R_2}{R_3} \qquad ...(7.3)$$

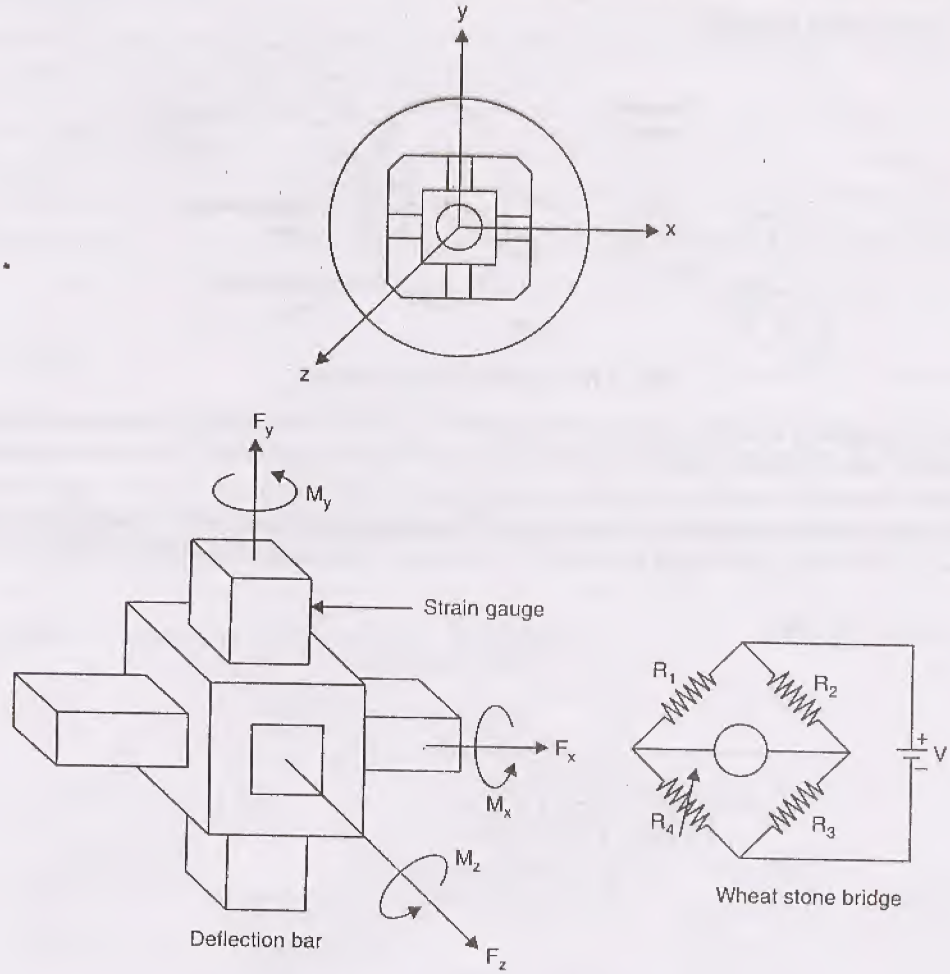


Fig. 7.9. Wrist Force/Torque Sensor.

To accommodate for the temperature changes one of the resistances in the Wheatstone bridge is made dummy. The needed performance specifications for the force sensors are

+ Linearity between response and applied force.
+ Low hysterisis and internal friction for restoring the original position, and to enhance the sensitivity.
+ Compact design to avoid the collision with other objects in the work space.
+ High stiffness to ensure the damping of the disturbing forces by higher natural frequency.

Encoders

There are two types encoders in use :

1. Incremental encoder that measures rotational speed and
2. The Linear encoders that measures linear speed.

- *Incremental Encoder :*

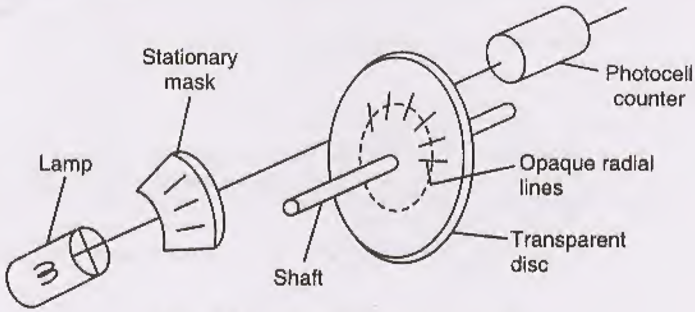


Fig. 7.10. (*a*) Incremental encoder.

An incremental encoder, Fig. 7.10 (*a*) senses the shaft rotation. A transparent disc with opaque radial lines (equally spaced) is mounted on the rotating shaft. The light coming from a lamp passes through the disc (rotating) and reaches the photocell counter that counts the passing of opaque lines crossing the light rays. The stationary mask with opaque lines of same spacing and thickness the signal produced is stronger. The pulse frequency is the measure of speed.

- *Linear Encoder :*

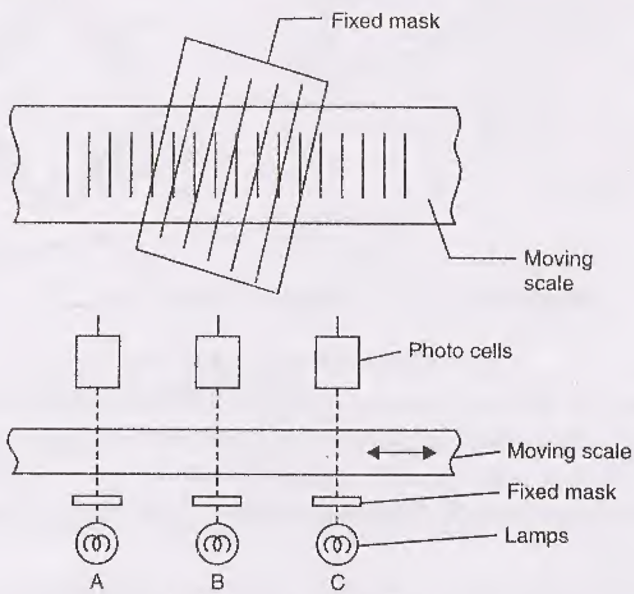


Fig. 7.10. (*b*) Linear encoder.

The moving scale with parallel opaque lines is mounted on the machine slide. A skewed fixed mask is kept in between the moving scale and the lamp. The light from the lamp passing through the moving opaque lines produces pulses sensed by photocells. The pulses are the measure of linear speed of the machine slide.

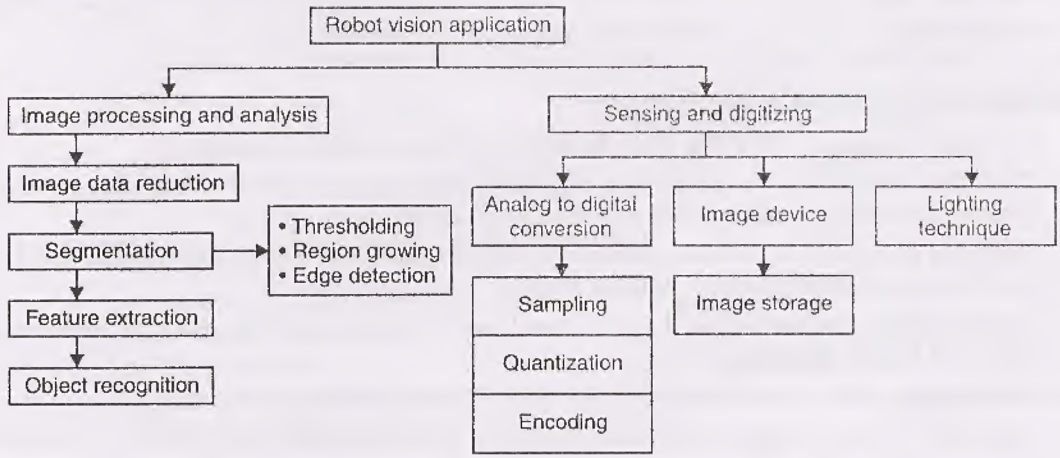
7.11 ROBOT VISION

The process of deriving, featurizing and analizing the details from the three dimensional object in the form of a picture is known as Robot vision or machine vision. As the application

utilizes the computer for processing it is also known as computer vision. The areas of processing and analysis of the images are categorized as follows :

Principal functions	*Functional Description*
• Sensing	+ The process that describes and gives out visual image.
• Preprocessting	+ Deals with the process of disturbance reduction and image development.
• Segmentation	+ Is associated with the technique of dividing the image into parts of need.
• Description	+ Distinguishes the parts of focus from other object for computation of the image feature.
• Recognition	+ Is the identifying stage where in parts like spanner, bolts and nuts are recognized.
• Interpretation	+ In this process the recognized objects are given useful meaning for task operation.

7.12 BLOCK DIAGRAM OF VISION SYSTEM



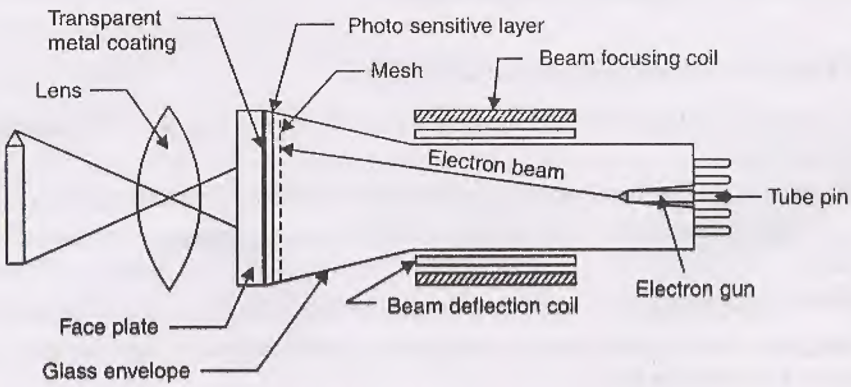


Fig. 7.11. Vidicon Camera.

7.13 CONSTRUCTIONAL FEATURES OF VIDICON CAMERA

Parts of Construction	*Relative Functions*
1. Lens	* Focuses the image of the object on to the camera.
2. Face plate	* Glass cover at front end of camera.
3. Transparent metal coating	* Acts as electrode which derives electrical video signal.
4. Photo sensitive layer	* Is a layer of resistive particles whose resistance is inversely proportional to light intensity.
5. Wire Mesh	* Decelerates the beam of electrons so that they reach photo sensitive layer with zero velocity.
6. Electron gun	* Generates beam of electrons that scans the photo sensitive layer.
7. Beam deflection coil	* Deflects the beam of electrons vertically and horizontally for scanning.
8. Beam focusing coil	* The electron beam is focused by this coil.
9. Tube pins	* Acts as connector to the electric supply source.
10. Glass envelope	* Provides a housing for above elements.

Working Principle of Vidicon Camera

- The metal coating of the faceplate is applied with a positive voltage.
- The photosensitive layer acts like a capacitor with negative charge on the inner surface and positive charge on the opposite side, as electron beam strikes.
- The light striking the photosensitive layer reduces the resistance and the current starts flowing and neutralizes the positive charge.
- As the image is formed on the target layer, the concentration of electrons is high in dark area and low in lighted area.
- The electrons so formed flow through metal layer and through the tube pins.
- Variation in current during the electron beam scanning motion produces a video signal proportional to the intensity of input image.

7.14 LIGHTING TECHNIQUE AND DEVICES

- The fundamental types of lighting devices used in robot vision are classified into the following groups :
  - (*a*) Diffuse surface Devices : are examplified by the flourescent lamps and lighted tables.
  - (*b*) Condensor projectors : transforms an diverging light source into a focusing light source.
  - (*c*) Flood or spot projector : used to illuminate object surface areas from all angles.
  - (*d*) Collimator : is a device which produces parallel beam of light on the object whose image is to be captured.

(*e*) Imagers : example slide projectors and optical enlargers produce at the object plane real form of an image.

The illumination techniques are many. Some such special cases are listed in table 7.1. along with the application. The objective of illumination is to provide a suitable environment for the camera to provide the realistic images of the object in the work-space.

There are basically two major type of illumination techniques : Front lighting and rear lighting. In the front lighting, source of light is on the same side of the camera and in rear lighting technique, the source of light is on the opposite side of the camera.

Table 7.1

Illumination Techniques

Front Light Source	
(a) Front illumination	* The feature of the image is defined by the surface flooded by light.
(b) Light field specular illumination	* Used for recognition of surface defects with light background.
(c) Dark field specular illumination	* Useful in recognition of surface defects in dark background.
(d) Front imager	* Super imposition of imaged light on object surface.

Back Light Source	
(a) Light field rear illumin-ation	* Used in simple measurements and inspection of the parts.
(b) Condensed rear illumination	* High contrast of images produced is useful in high magnification.
(c) Rear illumination collimator	* Parallel light ray source produced so that same plane objects are featured.
(d) Offset rear illumination.	* Highlights the object feature in transparent medium.

7.15 ANALOG TO DIGITAL CONVERSION

The imaging device like video camera gives analog signal voltage denoting the two dimensional images of the object. This information about the image has to be stored in the bit memory of the computer only on conversion to digital signal. The digital conversion of the analog signal is an approximation of reality with minimum error, done using a analog to digital (A/D) converter. This process has three staged phases :

(1) Sampling

(2) Quantization

(3) Encoding.

• Sampling

Let the function $f(x, y)$ denote the two-dimensional image pattern on the image device. The geometric co-ordinates x and y of the image plane are digitized to get information by the process known as 'image sampling'. After sampling the digitised function $f(x, y)$ in the spatial co-ordinates is generated which can be easily stored in the computer memory.

Assume

N = number of lines in the face plate of the image device like vidicon camera.

S = sampling capability of A/D converter in sec. (sampling capability is the cycle time or frequency needed to convert the analog signal of a pixel by the A/D converter)

R = scanning rate in second, for complete face plate.

R_d = line change over delay for the electron beam.

Hence the scanning rate per line,

$$R_L = \left(\frac{R}{N} + R_d\right) \qquad ...(7.4)$$

Number of pixels that can be processed/line,

$$P_n = \left(\frac{R + NR_d}{N.S}\right) \qquad ...(7.5)$$

Generally the scan line change over is given in the percentage of the scan rate for a line.

- **Quantization**

The digitization of the amplitude of the image function $f(x, y)$ depending on the intensity of the pixel is known as Quantization.

The number of quantization level,

$$Q = 2^n \qquad ...(7.6)$$

where the, n is number of memory bits in the A/D converter.

The quantization level spacing is given as

$$L = \frac{F_r}{Q} \qquad ...(7.7)$$

where F_r = full scale range of the camera in volts.

From equations (7.6) and (7.7)

$$L = \frac{F_r}{2^n} \qquad ...(7.8)$$

The digital approximation of the analog signal gives the error in quantization as

Quantization error, $$e_q = \pm\frac{1}{2}(L)$$

or $$e_q = \pm\frac{1}{2}\left(\frac{F_r}{2^n}\right) \qquad ...(7.9)$$

- **Encoding**

Depending on the image created on the faceplate of the camera, the intensity of the different pixel would be different. The conversion into the digital code of the amplitude levels follows the process of quantization. The digitized amplitude code is represented by the binary sequence of digits, which is known as encoding. The spaced quantization levels show the difference in intensities and the amplitude levels. All zeros in the binary sequence of digits represent dark (black) intensity level. All ones in the bit memory is the representative bright (white) intensity pixel. In between the two with combination of zeros and ones shows the gray color.

7.16 IMAGE STORAGE

The storage of image (digitized) in the computer memory is done by the frame buffer which can be made a part of the computer or frame grabber. The more popular frame grabber technique can be explained as follows.

The access and acquisition of the image is done in $\frac{1}{30}$ second by the frame grabber of a video camera. An average camera can produce a disturbance free data with a 6-bit buffer where as the quantization of the frame has a specification of 8 bits. The lower level bits are rejected in the operation of noise cleaning. As the resolution of the general human eye = $2^6(64)$, the image grabbed by the video camera would be sufficiently good.

The row and column counters of the frame grabber are synchronized by the electron beam of the camera. The signal sent by the computer to the position address (x, y) of the face-plate reads the information stored in the frame buffer, uniquely addressed by sampling and quantization.

7.17 IMAGE PROCESSING AND ANALYSIS

In the industrial applications the algorithms and programs are developed to process the images captured, digitized and stored in the computer memory. The size of data to be processed is huge, of the order of 10^6 which is to be substantially executed in $\frac{1}{30}$ seconds. The difficult and time consuming task of processing is handled effectively by the following techniques :

(1) Image data reduction

(2) Segmentation

(3) Feature extraction

(4) Object recognition.

• Image Data Reduction

The purpose of image data reduction is to reduce the volume of data either by ellimination of some or part processing, leading to the following sub-techniques.

(*a*) Digital conversion

(*b*) Windowing.

* *Digital conversion* is characterized by reduction in number of gray levels. For a 8-bit register each pixel would have 2^8 =256 gray levels. When fewer bits are used to represent pixel intensity the digital conversion is reduced, to suit the requirements.

The data reduction is effected in the following manner generalized as

Total number of bits on the face plate,

$$T_1 = N_r . N_c (2)^n \qquad ...(7.10)$$

where N_r = number of lines or rows

N_c = number of points per line

2^n = total gray levels.

Binary bit conversion for totally black and white intensities,

$$T_2 = N_c.N_r.(2)$$

Reduction in data volume $= (T_1 - T_2)$

$$= 2N_cN_r(2^{n-1} - 1) \quad ...(7.11)$$

* *Windowing* is processing a portion of the stored digital image. The portion of focus extracted for image processing is the window. A rectangular window is selected as to highlight the component of interest on the screen. The pixels of the faceplate within the window are processed and analized by the computer.

PROBLEMS

Example 7.1. *A rastor scan system of vision has a frame of face-plate with 256 lines, having $\frac{1}{3}$ sec. as the scanning rate. It may be assumed that the electron beam takes 10% of the scan time to move from one line to other line. If there are 256 pixels per line, determine the sampling rate.*

Sol. It is given in equation (7.5) as

$$P_n = \left(\frac{R + NR_d}{N.S}\right)$$

where $R = \frac{1}{3}$ sec.

$N = 256$ lines

$P_n = 256$ pixels

$R_d = 10\%$ of $\frac{R}{N} = \frac{0.1R}{N}$

Hence S = sampling rate in seconds/ pixel.

$$= \frac{R + 0.1R}{P_n.N} = \frac{1.1 \times \frac{1}{3}}{256 \times 256} = 5.6 \times 10^{-6} \text{ s/pixel.}$$

Example 7.2. *The maximum voltage range for a 8 bit capacity A/D converter is 18 V. Calculate the quantization levels, quantization level spacing, and the quantization error.*

Sol. From equation (7.6)

The number of quantization level,

$$Q = 2^n = 2^8 = 256$$

The quantization level spacing,

$$L = \frac{F_r}{Q}$$

where $F_r = 18$ volts

So

$$L = \frac{18}{256} = 0.0703 \text{ V}$$

The quantization error,

$$e_q = \pm\frac{1}{2}\,\mathrm{L} = \pm\,\frac{1}{2}\left(\frac{18}{256}\right) = 0.03515\ \mathrm{V}.$$

Segmentation

The representation different, distinct parts of the entire image data are grouped into area of similar characteristics is known as 'Segmentation'. The major segments of the images are regions and edges that differentiate them from the background. The image processing and analysis are explained by the following segmentation techniques.

(1) Edge detection

(2) Region growing

(3) Thresholding.

• *Edge detection*

At boundary the pixels on the faceplate different intensity levels which are stored in the computer in the binary form. This is the distinguishing feature of the object images. The features of similar region, at the edges show demarcation representing change over of the attributes. The edge detection is based on follow-the-edge procedure as shown in the Fig. 7.12. The procedure is to scan the pixel within the region, for which turn left and step or otherwise turnright and step from a starting point outside the boundary. This is continued till the end point meets the starting point.

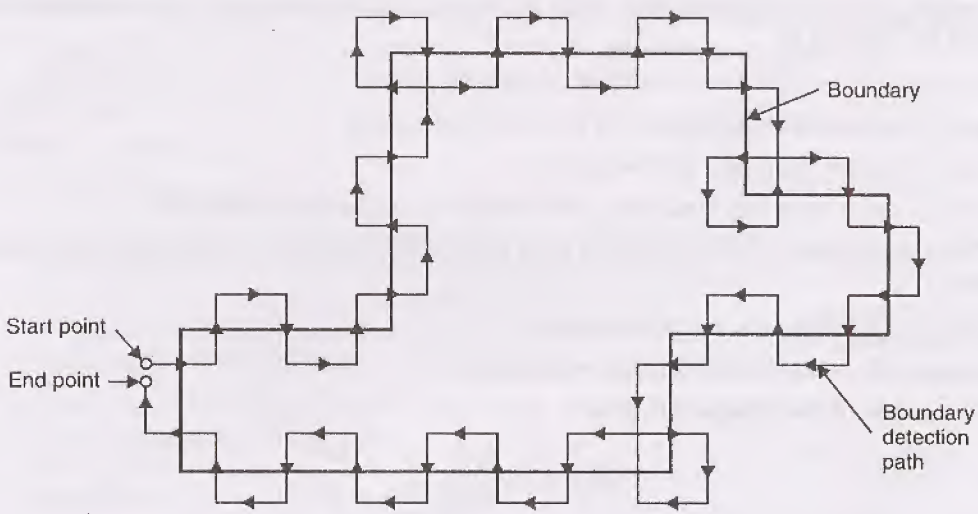
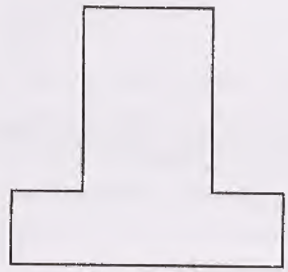


Fig. 7.12. Edge Detection.

• *Region growing* is the processing technique where grid elements processing similar attributes are grouped to form a region. The grid elements are collection of pixels which discretize the object image and the background formed on the faceplate. The properties and the spatial geometric co-ordinates of a region decide, on the process of merging or if to be left independent as a separate entity. The region growing procedure can be better understood by assigning 0s for the background and 1s for the object regions. This is depicted in the Fig. 7.13.

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0



Object image

Fig. 7.13. Region Growing Technique.

The region growing procedure can be briefed as follows :

- A pixel on the object is identified and assigned the value 1.
- The adjacent pixel are tracked for match in the attributes. The matching pixel is assigned 1 and non-matching pixel with 0.
- The terms are repeated till the complete screen is covered resulting in growth and identification of region.
- *Thresholding of image* is one of the principal object detection techniques when the binary image data to be processed is high in volume. Mathematically the threshold function can be defined as

$$T = T[s(x, y), p(x, y), f(x, y)]$$

where $s(x, y)$ = spatial co-ordinate of a point on the screen

$p(x, y)$ = local property at that point.

$f(x, y)$ = the intensity function of the image at the point of analysis.

The comparison of the threshold with that of the intensity decides the type of point on the plate.

when $f(x, y) > T$ $\quad s(x, y)$ is the object point

$f(x, y) < T$ $\quad s(x, y)$ is the background point.

The value of the image threshold,

$$g(x, y) = \begin{cases} 1 \text{ if } & f(x, y) > T \\ 0 \text{ if } & f(x, y) \le T \end{cases}$$

Dependability of T	Threshold type
on $f(x, y)$	global threshold
on $f(x, y)$ and $p(x, y)$	local threshold
on $s(x, y)$	dynamic threshold

7.18 FEATURE EXTRACTION

The images formed on the screen can have multiple objects which are to be distinguished from one another for processing and anylysis. The features that characterize uniquely, the objects provide means to extract the identification and comparison. This is accomplished by the features like area, diameter and perimeter, also minimum enclosing rectangle, and gray levels are considered in the feature extraction.

The area of the object is described by the region growing procedure as explained before. The area is given by

$$\text{Area} = \frac{(\text{perimeter})^2}{\text{thickness}},$$

where thickness = compactness of the object.

Diameter = (Thickness × Area).

The enclosing boundary that covers the specific area can be established by the pixel intensity difference, at the boundary.

The diameter of an object image is the maximum distance obtainable on two different points on the perimeter of an object.

An important observation is that the selected feature does not depend upon position and orientation of the boundary.

7.19 OBJECT RECOGNITION

One of the major approaches in image processing is the technique of matching the captured image with object to be recognized. The technique of object recognition is based on the feature extraction described previously. The powerful algorithms are used for this purpose, the techniques used which are

- Template Matching
- Structural Technique.
  - *Template matching* is a part of a general statistical pattern recognition technique. The object to be recognized is stored in computer memory in advance. The properties like area, perimeter etc., are calculated for the prototype pattern. Then the objects are matched and compared with the stored information, which is known as template matching. The success of this technique is dependent on less number of stored pattern.
  - *Structural technique* is technique deals with relation between features or the boundaries of an object which is sub-divided into primitive or the elements with defined inter-relations. This is a separate topic is known as syntactic pattern recognition, the explanation of which is beyond the scope of this book.

7.20 COMPONENTS OF DIGITAL IMAGE PROCESSING

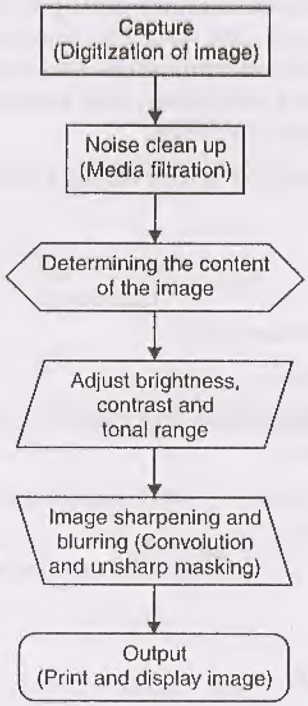


Fig. 7.14. Image Processing Block Diagram.

EXERCISE

7.1. With the help of a block diagram, explain the functions of a robotic vision system and devices used in the same. *(VTU–Jan./Feb. 2003)*

7.2. Write a short notes on object recognition technique. *(VTU–May/June 2004)*

7.3. Enlist the desirable sensor and transducer features. *(VTU–Jan./Feb. 2004)*

7.4. Give the classification of sensors with example. *(VTU–Jan./Feb. 2004)*

7.5. List the various sensors used in the robotic systems. *(VTU–May/June 2004)*

7.6. With a neat sketch explain the tactile sensors and the range sensors. *(VTU–May/June 2004)*

7.7. Sketch and explain a six component wrist sensors based on strain gauge element for force/torque sensing. *(VTU–May/June 2004)*

7.8. Discuss the desirable engineering features of sensors and transducers. *(VTU–Jan./Feb. 2003)*

7.9. Explain the types of touch sensors with neat sketches.

7.10. Explain principle and construction of Inductive type proximity sensors.

7.11. Explain the construction and operation of ultrasonic proximity sensors.

7.12. Explain with a neat sketch the optical proximity sensors.

7.13. Give the performance specifications for the force sensors.

7.14. Describe the principal functions of robot vision system.

7.15. Explain the construction and principle of operation of Vidicon camera.

7.16. What are the image devices used in robot lighting Technique ?

7.17. Explain the illumination technique used in robot vision system.

7.18. Explain sampling function of A/D conversion.
7.19. Write short notes on quantization and encoding.
7.20. Explain image storage.
7.21. Discuss digital conversion and windowing.
7.22. What is edge detection ? Explain the procedure.
7.23. Discuss region growing technique with example.
7.24. What is thresholding ? Explain threshold types.
7.25. Discuss feature extraction technique.
7.26. What is object recognition ? Explain template matching.
7.27. Explain with block diagram component of digital image processing.
7.28. State and explain the principle of Hall effect.
7.29. What are the advantages of Hall effect sensors ?
7.30. What is the function of encoder ? What are the types of encoders ?
7.31. Explain the principle of operation of incremental encoder.
7.32. Explain the working linear encoder.

8 Robot Programming

8.1 INTRODUCTION

For the robot user to get the task done from the robot manipulator there is the need for an effective and efficient communication method. Several subjective and objective communication techniques are developed to perform the task as determined to suit the application. The methods are : lead through teaching method, speech recognition and programming.

The lead through teaching which may be manual or powered is accomplished in the following steps.

- Leading the manipulator in slow motion controlled throughout the entire task operation.
- Storing the joint angles at needed path locations.
- Editing and play-back of the taught motion.
- Replay of the motion by the robot at a speed as specified by the user.

Speech recognition is voice technique where the user speaks out the discrete words of command. The stored commands are executed by the robot by word recognition process in real time leading to robot motion. The techniques is primitive and requires large storage space of memory to record speech data. This also needs extensive training for the user in developing a speech template.

Programming is the most general and versatile subjective method of human-robot communication.

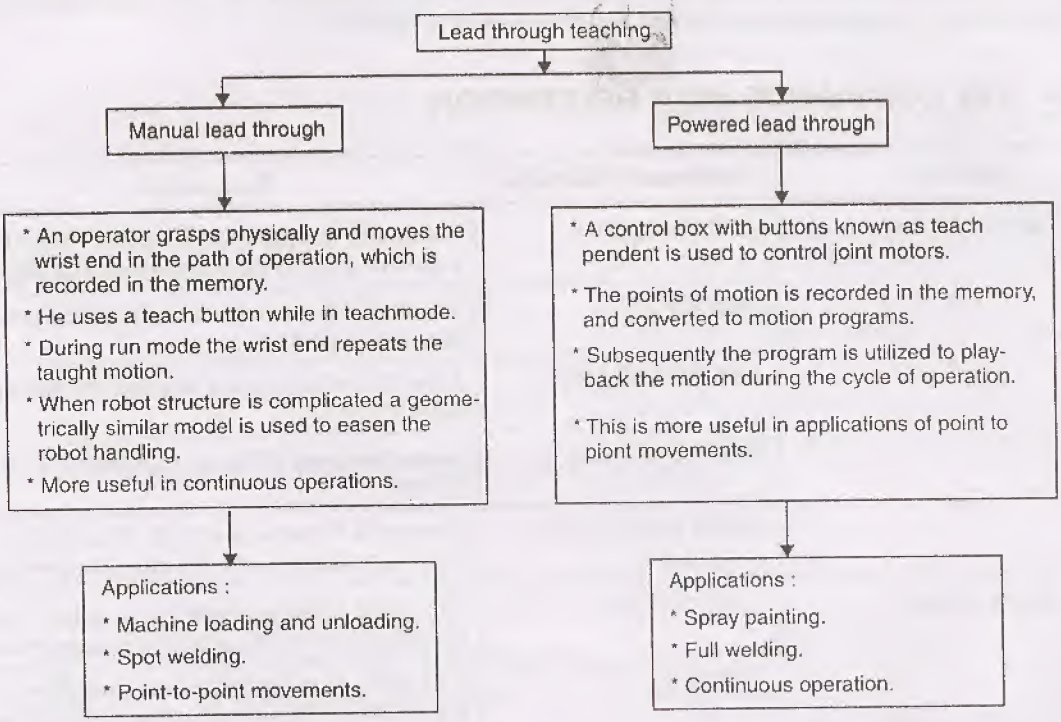
The considerations in robot programming are

- The three dimensional objects with different physical properties are to be manipulated.
- The environments of robot operations can be complex.
- Visualization of the object can be discrete.
- The processing and the analysis of the digital data from sensors and vision system has to be done in real time.

The high-level programming is oriented on tool and task basis, of operation sequence.

Robot Oriented Programming	*Object Oriented Programming*
• The sequence of robot motion is programmed and robot is made to follow. • The robot motions are explicitly defined. • The commands and actions are correspondingly related.	• The program developed on task basis as a sequence of goal positions. • The explicit definition of robot motion is not required. • No relation between command and action but the task is executed.

8.2 LEAD THROUGH PROGRAMMING



8.3 REQUIREMENT OF GOOD PROGRAMMING LANGUAGE (ROBOT ORIENTED)

• World Modeling

The set up of the work-space and the fixtures and feeders in which the parts are fixed are determined. Generally robots and the objects are confined to well defined world-space. The positional uncertainty is minimized by using restricted feeders and fixtures.

• Position Specification

The position and orientation of the parts are defined interms of co-ordinate frame. The feeder and beam locator and their features are described by using data structures provided by the language.

• Motion Specification

The general PNP activity is divided into the sequence of action such as movement of the arm from a initial configuration to grasp position, picking up an object and moving to the final configuration.

• Sensory Control

To take care of the uncertainty in location and dimension of the objects in the work-envelope sensing is to be performed. The information gathered by the sensors acts as feedback from environment.

• **Programming Support**

The programming supports like editing and debugging etc are provided by the sophisticated programming platform for the user to program.

8.4 VAL COMMANDS WITH DESCRIPTION

Definition	*Command Statement*	*Explanation*
1. Motion control	APPRO P1, Z1	Command to approach the point P1 in the z-direction by Z1 distance above the object.
	MOVE P1	Command to move the arm from the present position to point P1.
	MOVE P1 VIA P2	Asks the robot to move to point P1 through point P2.
	DMOVE (J1, ΔX)	Moves the joint J1 by an increment of ΔX (linear)
	DMOVE (J1, J2, J3) ($d\alpha$, $d\beta$, $d\theta$)	Command to move joints J1, J2 and J3 by incremental angles of $d\alpha$, $d\beta$, $d\theta$ respectively.
2. Speed control	SPEED V IPS	The speed of the end effector is to be V inch per second at the time of program execution.
	SPEED R	Command to operate the arm end effector at R percent of the normal speed at the time of program execution.
3. Position control	HERE P1	Defining the name of a point as P1.
	DEFINE P1 = POINT (x, y, z, w_α, w_β, w_θ)	The command defines the point P1 with x, y, z co-ordinates and w_α, w_β, w_θ the wrist rotation angles.
	Path control : DEFINE PATH 1 = PATH (P1, P2, P3)	The path of the end effector is defined by the connection between points P1, P2 and P3 in series.
	MOVE PATH 1	Movement of the end effector along path 1.
	Frame definition: DEFINE FRAME 1 = FRAME (P1, P2, P3)	— Assings variable name to FRAME 1 defined by points P1, P2 and P3. P1-origin, P2-point along x-axis and P3-point along xy plane.
	MOVE ROUTE: FRAME 1.	— Defines the movement in the path for frame 1.
4. End effector operation	OPEN	— Opens the gripper fingers.
	CLOSE 50 MM	— In forms gripper to close keeping 50 mm width between the fingers.

	CLOSE 5 LB.	— Applies 5 Lb gripper force.
	CENTER	— Closes the gripper slowly till the establishment of contact with the object to be gripped.
	OPERATE TOOL (SPEED N RPM)	— Positioning and operating the powered tool. Here the EE is replaced by servo powered tool.
5. Operation of the sensors	SIGNAL 4, ON	The command actuates the output port 4 and turns on at certain stage of the program.
	SIGNAL 5, OFF. WAIT 13, ON	The output port 5 is turned off. The device gives a feed back signal indicating that it is on.
	REACT 16, SAFETY.	The change in signal (if any), in the input line 16, should be deviated to the sub-routine SAFETY.

8.5 DEFINITION AND STATEMENTS OF AL AND AML

Definition	*AL-statement*	*AML-statement*
• Definition of base frame	FRAME (ROT(Z, Angle), VECTOR (x, y, z))	frame<<x, y, z>, EULER ROT (<ROTx, ROTy, ROTz>)>
• Definition of feature frame	base * TRANS(ROT(x, angle), Vector (x, y, z) ;	DOT (base, <<x, y, z>, EULER ROT (<ROTx, Roty, Rotz>)>) ;
• Definition of motion statement	MOVE barm to frame A ;	MOVE (<joint i, joint j>, <distance, angle>) ;
• Sensing control statement	OPEN bhand to distance ;	MOVE (gripper, distance) ;
• Force sensing and compliance statement	WITH FORCE (Z) = force Z WITH FORCE (X) = force X WITH FORCE (Y) = force in Y WITH DURATION = time in sec.	Fmons = MONITOR (<SLP, SRP>, X, O, R) ; SLP and SRP are force sensors.

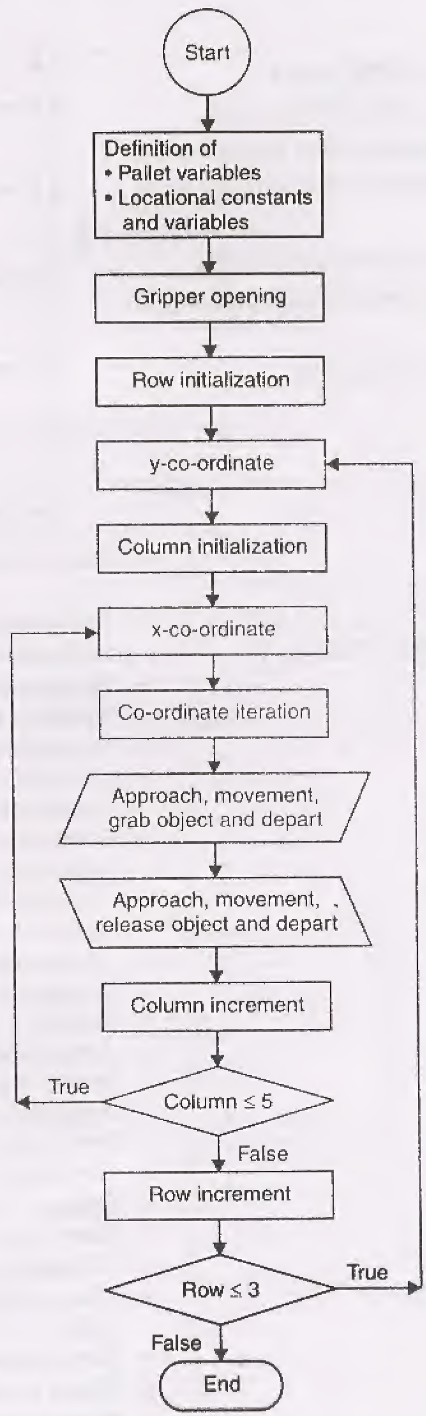
8.6 PROGRAMMING LANGUAGES-FEATURES AND APPLICATION

Language ⇒ / ⇓ *Feature*	*AL*	*AML*	*RPL*	*VAL*
• Language base	Concurrent, Pascal	LISP, Pascal	FORTRAN, LISP	BASIC
• Level orientation	Robot, Object	Robot	Robot	Robot
• Motion specification	Frame	Joints	Joints	Joints, Frame
• Sensing control	Position, Force	Position	Position, Vision	Position, Force
• Robot multiplicity	YES	NO	NO	NO
• Robot application	PUMA—stanford arm	IBM arm	PUMA	PUMA

8.7 PROGRAM FOR PNP (PICK AND PLACE) ACTIVITY

VAL STATEMENT	*Statement Description*
BRANCH PICK	— The branch of program indicating part pick.
MOVE INTER	— Move to a intermediate position above chute.
WAIT 12	— Wait for a incoming part to the chute.
SIGNAL 5	— Open gripper fingers (sensor control).
MOVE PICK-UP	— Move gripper and pick-up the object.
SIGNAL 6	— Close the gripper to grab the object.
MOVE INTER	— Depart to intermediate position above chute.
END BRANCH	— End of pick-up activity.
BRANCH PLACE	— Start of placing activity.
MOVE Z (–50)	— Position part and gripper above the pallet.
SIGNAL 5	— Open gripper to release the part.
MOVE Z (50)	— Depart from the place (pallet) point.
END BRANCH	— End of place activity.

8.8 FLOW CHART FOR PALLETIZATION PROGRAM



8.9 PROGRAM TO PALLETISE THE OBJECT

Given Data

Pallet variables

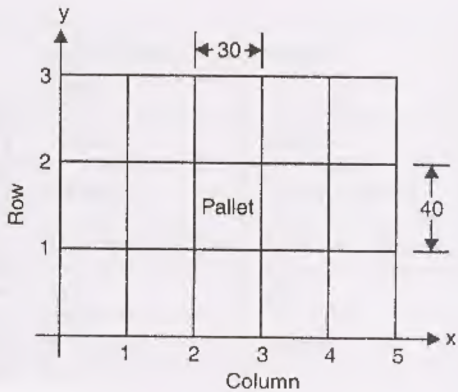
ROW	integer ROW count.
COLUMN	integer COLUMN count.
X	co-ordinate value along x-axis.
Y	co-ordinate value along y-axis.

Locational constants

PICK-UP	point of pick-up of objects.
CORNER	start of point in the pallet.

Locational variable

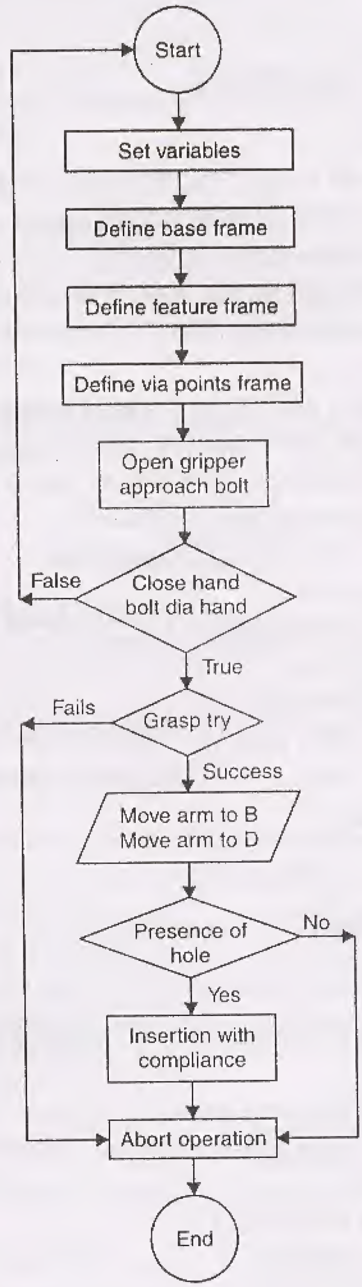
DROP	point of dropping.



Palletising program.

VAL STATEMENT	*Description of Statement*
PROGRAM PALLETISE	— Start of program.
DEFINE PICK-UP = JOINTS (1, 2, 3, 4, 5)	
DEFINE CORNER = JOINTS (1, 2, 3, 4, 5)	— Definition of constants of location.
DEFINE DROP = CO-ORDINATES (X, Y)	— Definition of variable of pallets.
OPEN	— Gripper action control to open fingers.
ROW = 0.0	— Initialise the row count.
10 Y= ROW * 40.0	— Computation of dropping point (Y-co-ordinate)
COLUMN = 0.0	— Initialization of column count.
20 X = COLUMN * 30.0	— Computation of drop off point (X-co-ordinate)
DROP = CORNER + (X, Y)	— Defining drop for every iteration.
APPRO PICK-UP, 50	— Approach above pallet for pick-up.
MOVE PICK-UP	— Positioning for pick-up action.
CLOSE	— Gripper action control to close the fingers.
DEPART 50	— Action sequence for arm to move away from the pick-up point.
APPRO DROP, 50	— Approach command to position above drop point.
MOVES DROP	— Command to drop object.
OPEN	— Open the gripper fingers.
DEPART 50	— Move away from drop point.
COLUMN = COLUMN + 1.	— Column increment.
IF COLUMN LT 5 GO TO 20	— Check for the column limit (LT ⇒ less than)
ROW = ROW + 1	— Row increment.
IF ROW LT 3 GO TO 10	— Check for ROW limit.
END PROGRAM	— End of program.

8.10 FLOW CHART FOR BOLT INSERTION PROGRAM



8.11 AL PROGRAM FOR BOLT INSERTION TASK

```
BEGIN insertion
bolt_dia ← 12 * mm ;
bolt_height ← 25 * mm ;                                Set Variables.
tries ← 0 ;
grasped ← false
beam ← FRAME (ROT(Z, 90 * deg), VECTOR (500, 375,0) * mm) ;     Def. Base frame.
feeder ← FRAME (nilrot, VECTOR(625, 500,0) * mm) ;
bolt_grasp ← feeder * TRANS (nilrot, nilvect) ;
bolt_tip ← bolt_grasp * TRANS (nilrot, VECTOR(0, 0, 12) * mm) ;
beam_bore ← beam * TRANS (nilrot, VECTOR(0, 0, 25) * mm) ;
                                                          Def. feature frame.
A ← feeder * TRANS (nilrot, VECTOR(0, 0, 125) * mm) ;
B ← feeder * TRANS (nilrot, VECTOR(0, 0, 200) * mm) ;      Def. of via point frame.
C ← beam_bore * TRANS (nilrot, VECTOR(0, 0, 125) * mm) ;
D ← beam_bore * TRANS (nilrot, bolt_height * Z) ;
OPEN bhand To bolt_dia + 25 * mm ; }   Open hand.
MOVE barm To bolt_grasp VIA A.
WITH APPROACH = –Z WRT feeder ;         Position hand above bolt.
CLOSE bhand To 0.9 * bolt_dia ;
IF bhand < bolt_dia THEN BEGIN ;
OPEN bhand To bolt_dia + 25 * mm ;
MOVE barm To 0 – 1 * Z * mm ;          Bolt grasp activities statements.
END ELSE grasped ← true ;
tries ← tries + 1 ;
UNTIL grasped OR (tries > 4) ;
IF NOT grasped THEN ABORT ("bolt grasp failed") ;
MOVE barm To B
VIA A
WITH DEPARTURE = Z WRT feeder ;          Movement of the arm to B and D.
MOVE barm To D VIA C
WITH APPROACH = –Z WRT beam_bore ;
MOVE barm to 0 – 0.1 * Z * mm ON FORCE (Z) > 200 * gms
DO ABORT ("Hole absent") ;                                  Check for Hole.
MOVE barm To beam-bore DIRECTLY
WITH FORCE (Z) = –200 * gms
WITH FORCE (X) = 0 gms                Execution of insertion with compliance.
WITH FORCE (Y) = 0 gms
WITH DURATION = 5 * seconds.
END insertion.
```

8.12 PROGRAM OPERATING AND STORAGE

The defining and usage of the operating system in robot programming is in similar lines with computer programming languages. The purpose of having operating system is to maximise the performance and the efficiency of computer systems and the peripheral devices like CRT monitor, alphanumeric key-board, printers and storage devices. The devices used to store the program are magnetic tapes or disks.

• Definition

The operating system is a computer software that allows the user to carry out the mechanisms of

— Writing a new program
— Editing the existing program
— Executing a given program
— Perform other functions like compiling.

• Modes of Operating System

<table>
<tr><td colspan="2">Operating system</td></tr>
<tr><td colspan="2">Monitor mode</td></tr>
<tr><td>Edit Mode</td><td>Run Mode</td></tr>
</table>

8.13 MODES OF OPERATING SYSTEM

Functions of Monitor Mode

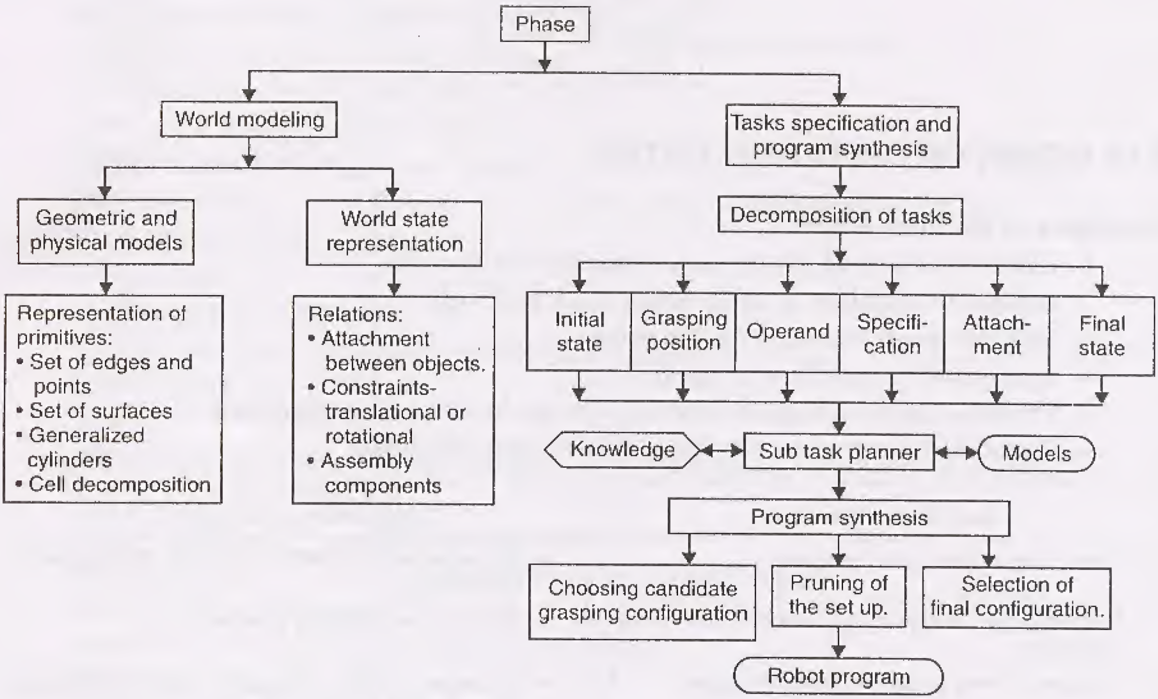
- Establishes overall supervisory control of the systems.
- Defines the location in space using teach pendant.
- Sets the speed of control for the robot.
- Storing of the functional program.
- Transferring the program between storage devices and control memory.
- Co-ordinating other modes like run mode and edit mode.

Run Mode Functions	*Edit Mode Functions*
• Execution of the robot program.	• Provedes sets of instructions for writing new programs.
• Performing sequence of instruction in the program.	• Editing the existing program.
• Testing of new program and debugging.	• Deletion and changing the existing instructions to the program.
• Display of the error messages during execution.	• Insertion of new lines into the program.
• Change over to the edit mode for correction.	• The editing of a program is language dependent.
• Shutting between edit and run mode to write new program while execution of a program.	

8.14 COMMANDS IN MONITOR MODE

Commands	*Functional Description*
1. EDIT (Program name)	→ Allows writing a new program or editing the existing program.
2. EXIT	→ Stores the program into control memory and return to edit mode.
3. STORE (Program name)	→ Stores the program into permanent storage device (disk or tape).
4. READ (Program name)	→ Reads a file from the storage into the control memory.
5. LIST (Program name)	→ Displays the file on monitor.
6. DIRECTORY	→ Lists the program files stored.
7. PRINT (Program name)	→ The program print on hard copy.
8. DELETE ERASE (Program name)	→ The deletion of the program from memory or storage.
9. EXECUTE, EX or EXEC (program name)	→ Instructs the robot to execute the program.
10. ABORT or STOP	→ Discontinues the program execution and stops robot motion.

8.15 ROBOT TASK PLANNING



EXERCISE

8.1. Explain the different ways by which robot teaching can be performed. *(VTU–Jan./Feb. 2003)*

8.2. Explain the following :

(*i*) Manual programming

(*ii*) Lead through teaching *(VTU–May/June 2004)*

8.3. What are the critical informations required for task programming of robot ? *(VTU–Jan./Feb. 2004)*

8.4. Discuss the programming methods used in robots mentioning their specific field of application. *(VTU–Jan./Feb. 2004)*

8.5. Write a robot program for PNP activity. *(VTU–Jan./Feb. 2004)*

8.6. Write a VAL program for inserting an OBJECT into a box by approaching 45 mm above the object by moving to an intermediate point 'P' along a straight line and approaching the box 70 mm from above and finally setting the gripper 25 mm above the box. Show the end effector path diagram. *(VTU–May/June 2004)*

8.7. Write a program in VAL for palletization of parts in a PALLET having 4 rows that are 50 mm apart and 6 columns 40 mm apart. The robot must pick parts from an incoming chute and are 25 mm tall. Use in the program the following names for variables ROW, COLUMN, X and Y, and use names for location constants PICK-UP, CORNER, and DROP. *(VTU–Jan./Feb. 2003)*

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